Chapter 6

Materials selection – case studies

6.1 Introduction and synopsis

Here we have a collection of case studies* illustrating the screening methods** of Chapter 5. Each is laid out in the same way:

(a) *the problem statement,* setting the scene;
(b) *the model,* identifying function, objectives and constraints from which emerge the property limits and material indices;
(c) *the selection* in which the full menu of materials is reduced by screening and ranking to a short-list of viable candidates; and
(d) *the postscript,* allowing a commentary on results and philosophy.

Techniques for seeking further information are left to later chapters.

The first few examples are simple but illustrate the method well. Later examples are less obvious and require clear identification of the objectives, the constraints, and the free variables. Confusion here can lead to bizarre and misleading conclusions. Always apply common sense: does the selection include the traditional materials used for that application? Are some members of the subset obviously unsuitable? If they are, it is usually because a constraint has been overlooked: it must be formulated and applied.

The case studies are deliberately simplified to avoid obscuring the method under layers of detail. In most cases nothing is lost by this: the best choice of material for the simple example is the same as that for the more complex, for the reasons given in Chapter 5.

6.2 Materials for oars

Credit for inventing the rowed boat seems to belong to the Egyptians. Boats with oars appear in carved relief on monuments built in Egypt between 3300 and 3000 BC. Boats, before steam power, could be propelled by poling, by sail and by oar. Oars gave more control than the other two, the military potential of which was well understood by the Romans, the Vikings and the Venetians.

* A computer-based exploration of these and other case studies can be found in *Case Studies in Materials Selection* by M.F. Ashby and D. Cebon, published by Granta Design, Trumpington Mews, 40B High Street, Trumpington CB2 2LS, UK (1996).

**The material properties used here are taken from the *CMS* compilation published by Granta Design. Trumpington Mews, 40B High Street, Trumpington CB2 2LS, UK.

Records of rowing races on the Thames in London extend back to 1716. Originally the competitors were watermen, rowing the ferries used to carry people and goods across the river. Gradually gentlemen became involved (notably the young gentlemen of Oxford and Cambridge), sophisticating both the rules and the equipment. The real stimulus for development of boat and oar came in 1900 with the establishment of rowing as an Olympic sport. Since then both have exploited to the full the craftsmanship and materials of their day. Consider, as an example, the oar.

The model

Mechanically speaking, an oar is a beam, loaded in bending. It must be strong enough to carry the bending moment exerted by the oarsman without breaking, it must have just the right stiffness to match the rower's own characteristics and give the right 'feel', and — very important — it must be as light as possible. Meeting the strength constraint is easy. Oars are designed on stiffness, that is, to give a specified elastic deflection under a given load. The upper part of Figure 6.1 shows an oar: a blade or 'spoon' is bonded to a shaft or 'loom' which carries a sleeve and collar to give positive location in the rowlock. The lower part of the figure shows how the oar stiffness is measured: a 10 kg weight is hung on the oar 2.05 m from the collar and the deflection at this point is measured. A soft oar will deflect nearly 50 mm; a hard one only 30. A rower, ordering an oar, will specify how hard it should be.

The oar must also be light; extra weight increases the wetted area of the hull and the drag that goes with it. So there we have it: an oar is a beam of specified stiffness and minimum weight. The material index we want was derived in Chapter 5 as equation (5.11). It is that for a light, stiff beam:

$$M = \frac{E^{1/2}}{\rho} \tag{6.1}$$

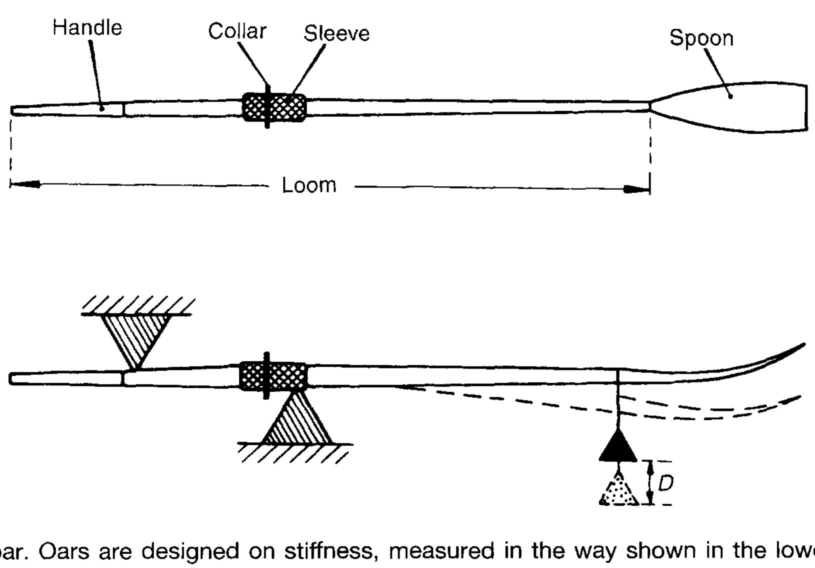

Fig. 6.1 An oar. Oars are designed on stiffness, measured in the way shown in the lower figure, and they must be light.

Table 6.1 Design requirements for the oar

Function	Oar, meaning light, stiff beam
Objective	Minimize the mass
Constraints	(a) Length L specified (b) Bending stiffness S specified (c) Toughness $G_c > 1\,kJ/m^2$ (d) Cost $C_m < \$100/kg$

There are other obvious constraints. Oars are dropped, and blades sometimes clash. The material must be tough enough to survive this, so brittle materials (those with a toughness less than $1\,kJ/m^2$) are unacceptable. And, while sportsmen will pay a great deal for the ultimate in equipment, there *are* limits on cost. Given these requirements, summarized in Table 6.1, what materials should make good oars?

The selection

Figure 6.2 shows the appropriate chart: that in which Young's modulus, E, is plotted against density, ρ. The selection line for the index M has a slope of 2, as explained in Section 5.3; it is positioned so that a small group of materials is left above it. They are the materials with the largest values of M, and it is these which are the best choice, provided they satisfy the other constraints (simple property limits on toughness and cost). They contain three classes of material: woods, carbon and glass-fibre reinforced polymers, and certain ceramics (Table 6.2). Ceramics are brittle; their toughnesses fail to meet that required by the design. The recommendation is clear. Make your oars out of wood or, better, out of CFRP.

Postscript

Now we know what oars *should* be made of. What, in reality, is used? Racing oars and sculls are made either of wood or of a high performance composite: carbon-fibre reinforced epoxy.

Wooden oars are made today, as they were 100 years ago, by craftsmen working largely by hand. The shaft and blade are of Sitka spruce from the northern US or Canada, the further north the better because the short growing season gives a finer grain. The wood is cut into strips, four of which are laminated together (leaving a hollow core) to average the stiffness. A strip of hardwood is bonded to the compression side of the shaft to add stiffness and the blade is glued to the shaft. The rough oar is then shelved for some weeks to settle down, and finished by hand cutting and polishing. The final spruce oar weighs between 4 and 4.3 kg, and costs (in 1998) about £150 or $250.

Composite blades are a little lighter than wood for the same stiffness. The component parts are fabricated from a mixture of carbon and glass fibres in an epoxy matrix, assembled and glued. The advantage of composites lies partly in the saving of weight (typical weight: 3.9 kg) and partly in the greater control of performance: the shaft is moulded to give the stiffness specified by the purchaser. Until recently a CFRP oar cost more than a wooden one, but the price of carbon fibres has fallen sufficiently that the two cost about the same.

Could we do better? The chart shows that wood and CFRP offer the lightest oars, at least when normal construction methods are used. Novel composites, not at present shown on the chart, might permit further weight saving; and functional-grading (a thin, very stiff outer shell with a low density core) might do it. But both appear, at present, unlikely.

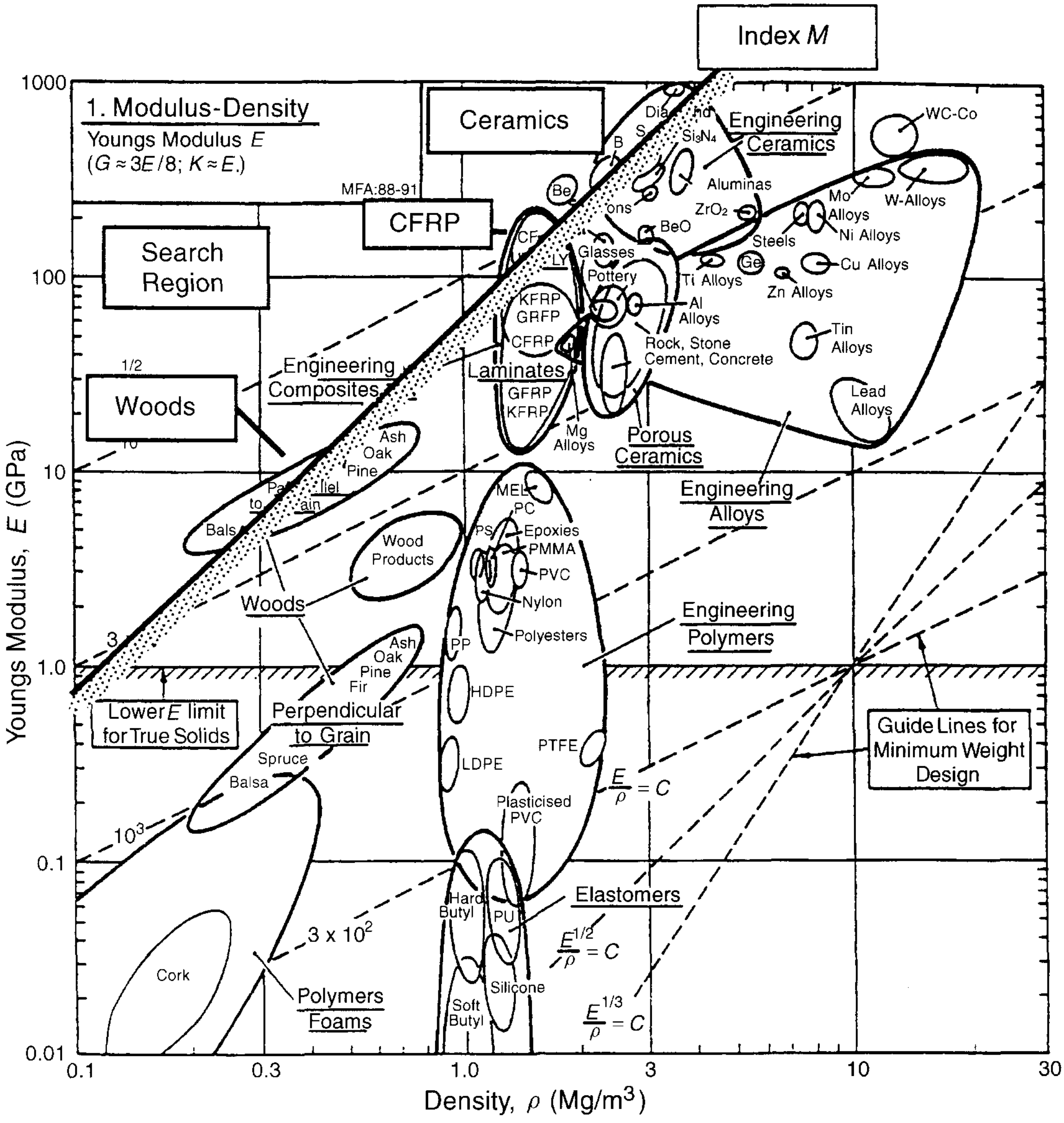

Fig. 6.2 Materials for oars. CFRP is better than wood because the structure can be controlled.

Table 6.2 Materials for oars

Material	*M* $(GPa)^{1/2}/(Mg/m^3)$	*Comment*
Woods	5–8	Cheap, traditional, but with natural variability
CFRP	4–8	As good as wood, more control of properties
GFRP	2–3.5	Cheaper than CFRP but lower *M*, thus heavier
Ceramics	4–8	Good *M* but toughness low and cost high

Further reading

Redgrave, S. (1992) *Complete Book of Rowing*, Partridge Press, London.

Related case studies

Case Study 6.3: Mirrors for large telescopes
Case Study 6.4: Table legs

6.3 Mirrors for large telescopes

There are some very large optical telescopes in the world. The newer ones employ complex and cunning tricks to maintain their precision as they track across the sky — more on that in the Postscript. But if you want a simple telescope, you make the reflector as a single rigid mirror. The largest such telescope is sited on Mount Semivodrike, near Zelenchukskaya in the Caucasus Mountains of Russia. The mirror is 6 m (236 inches) in diameter. To be sufficiently rigid, the mirror, which is made of glass, is about 1 m thick and weighs 70 tonnes.

The total cost of a large (236-inch) telescope is, like the telescope itself, astronomical — about UK £150 m or US $240 m. The mirror itself accounts for only about 5% of this cost; the rest is that of the mechanism which holds, positions and moves it as it tracks across the sky. This mechanism must be stiff enough to position the mirror relative to the collecting system with a precision about equal to that of the wavelength of light. It might seem, at first sight, that doubling the mass m of the mirror would require that the sections of the support structure be doubled too, so as to keep the stresses (and hence the strains and displacements) the same; but the heavier structure then deflects under its own weight. In practice, the sections have to increase as m^2, and so does the cost.

Before the turn of the century, mirrors were made of speculum metal (density: about 8 Mg/m^3). Since then, they have been made of glass (density: 2.3 Mg/m^3), silvered on the front surface, so none of the optical properties of the glass are used. Glass is chosen for its mechanical properties only; the 70 tonnes of glass is just a very elaborate support for 100 nm (about 30 g) of silver. Could one, by taking a radically new look at materials for mirrors, suggest possible routes to the construction of lighter, cheaper telescopes?

The model

At its simplest, the mirror is a circular disc, of diameter $2a$ and mean thickness t, simply supported at its periphery (Figure 6.3). When horizontal, it will deflect under it own weight m; when vertical it will not deflect significantly. This distortion (which changes the focal length and introduces aberrations into the mirror) must be small enough that it does not interfere with performance; in practice, this means that the deflection δ of the midpoint of the mirror must be less than the wavelength of light. Additional requirements are: high dimensional stability (no creep), and low thermal expansion (Table 6.3).

The mass of the mirror (the property we wish to minimize) is

$$m = \pi a^2 t \rho \tag{6.2}$$

where ρ is the density of the material of the disc. The elastic deflection, δ, of the centre of a horizontal disc due to its own weight is given, for a material with Poisson's ratio of 0.3 (Appendix A: 'Useful

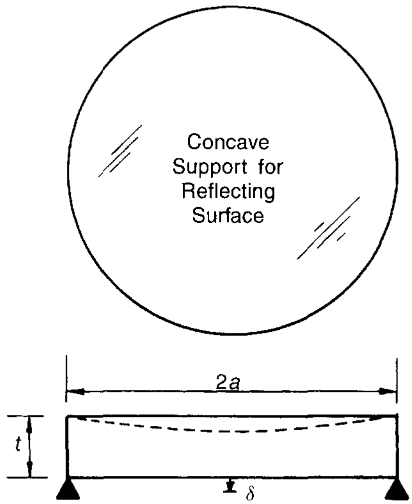

Fig. 6.3 The mirror of a large optical telescope is modelled as a disc, simply supported at its periphery. It must not sag by more than a wavelength of light at its centre.

Table 6.3 Design requirements for the telescope mirror

Function	Precision mirror
Objective	Minimize the mass
Constraints	(a) Radius a specified (b) Must not distort more than δ under its own weight (c) High dimensional stability: no creep, no moisture take-up, low thermal expansion

Solutions'), by

$$\delta = \frac{3}{4\pi}\frac{mga^2}{Et^3} \tag{6.3}$$

The quantity g in this equation is the acceleration due to gravity: 9.81 m/s^2; E, as before, is Young's modulus. We require that this deflection be less than (say) 10 μm. The diameter of the disc is specified by the telescope design, but the thickness is a free variable. Solving for t and substituting this into the first equation gives

$$m = \left(\frac{3g}{4\delta}\right)^{1/2} \pi a^4 \left[\frac{\rho}{E^{1/3}}\right]^{3/2} \tag{6.4}$$

The lightest mirror is the one with the greatest value of the material index

$$\boxed{M = \frac{E^{1/3}}{\rho}} \tag{6.5}$$

We treat the remaining constraints as property limits, requiring a melting point greater than 1000 K to avoid creep, zero moisture take up, and a low thermal expansion coefficient ($\alpha < 20 \times 10^{-6}$/K).

The selection

Here we have another example of elastic design for minimum weight. The appropriate chart is again that relating Young's modulus E and density ρ — but the line we now construct on it has a slope of 3, corresponding to the condition $M = E^{1/3}/\rho = \text{constant}$ (Figure 6.4). Glass lies on the line $M = 2\,(\text{GPa})^{1/3}\text{m}^3/\text{Mg}$. Materials which lie above it are better, those below, worse. Glass is much better than steel or speculum metal (that is why most mirrors are made of glass); but it is less

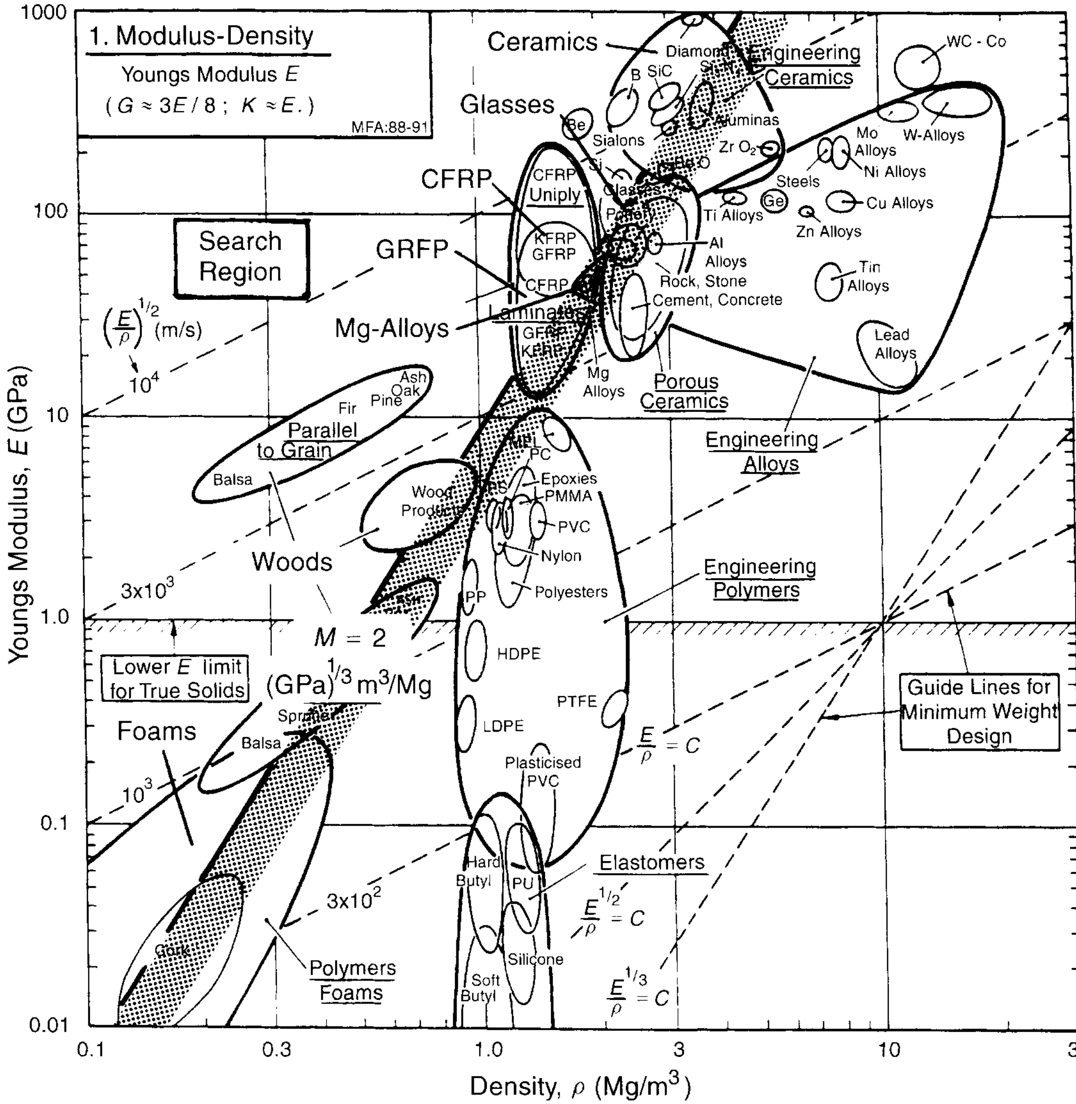

Fig. 6.4 Materials for telescope mirrors. Glass is better than most metals, among which magnesium is a good choice. Carbon-fibre reinforced polymers give, potentially, the lowest weight of all, but may lack adequate dimensional stability. Foamed glass is a possible candidate.

Table 6.4 Mirror backing for 200-inch telescope

Material	$M = E^{1/3}/\rho$ $(GPa)^{1/3}m^3/Mg$	*m (tonne)* $a = 6$ *m*	*Comment*
Steel (or Speculum)	0.7	158	Very heavy. The original choice.
Concrete	1.4	56	Heavy. Creep, thermal distortion a problem.
Al-alloys	1.5	53	Heavy, high thermal expansion.
Glass	1.6	48	The present choice.
GFRP	1.7	44	Not dimensionally stable enough — use for radio telescope.
Mg-alloys	2.1	38	Lighter than glass but high thermal expansion.
Wood	3.6	14	Dimensionally unstable.
Beryllium	3.65	14	Very expensive — good for small mirrors.
Foamed polystyrene	3.9	13	Very light, but dimensionally unstable. Foamed glass?
CFRP	4.3	11	Very light, but not dimensionally stable; use for radio telescopes.

good than magnesium, several ceramics, carbon-fibre and glass-fibre reinforced polymers, or — an unexpected finding — stiff foamed polymers. The shortlist before applying the property limits is given in Table 6.4.

One must, of course, examine other aspects of this choice. The mass of the mirror can be calculated from equation (6.5) for the materials listed in the table. Note that the polystyrene foam and the CFRP mirrors are roughly one-fifth the weight of the glass one, and that the support structure could thus be as much as 25 times less expensive than that for an orthodox glass mirror. But could they be made?

Some of the choices — the polystyrene foam or the CFRP — may at first seem impractical. But the potential cost saving (the factor of 25) is so vast that they are worth examining. There are ways of casting a thin film of silicone rubber or of epoxy onto the surface of the mirror-backing (the polystyrene or the CFRP) to give an optically smooth surface which could be silvered. The most obvious obstacle is the lack of stability of polymers — they change dimensions with age, humidity, temperature and so on. But glass itself can be reinforced with carbon fibres; and it can also be foamed to give a material with a density not much greater than polystyrene foam. Both foamed and carbon-reinforced glass have the same chemical and environmental stability as solid glass. They could provide a route to large cheap mirrors.

Postscript

There are, of course, other things you can do. The stringent design criterion ($\delta > 10\,\mu m$) can be partially overcome by engineering design without reference to the material used. The 8.2 m Japanese telescope on Mauna Kea, Hawaii and the Very Large Telescope (VLT) at Cerro Paranal Silla in Chile each have a thin glass reflector supported by little hydraulic or piezo-electric jacks that exert distributed forces over its back surface, controlled to vary with the attitude of the mirror. The Keck telescope, also on Mauna Kea, is segmented, each segment independently positioned to give optical focus. But the limitations of this sort of mechanical system still require that the mirror meet a stiffness target. While stiffness at minimum weight is the design requirement, the material-selection criteria remain unchanged.

Radio telescopes do not have to be quite as precisely dimensioned as optical ones because they detect radiation with a longer wavelength. But they are much bigger (60 metres rather than 6) and they suffer from similar distortional problems. Microwaves have wavelengths in the mm band, requiring precision over the mirror face of 0.25 mm. A recent 45 m radio telescope built for the University of Tokyo achieves this, using CFRP. Its parabolic surface is made of 6000 CFRP panels, each servo controlled to compensate for macro-distortion. Recent telescopes have been made from CFRP, for exactly the reasons we deduced. Beryllium appears on our list, but is impractical for large mirrors because of its cost. Small mirrors for space applications must be light for a different reason (to reduce take-off weight) and must, in addition, be as immune as possible to temperature change. Here beryllium comes into its own.

Related case studies

Case Study 6.5: Materials for table legs
Case Study 6.20: Materials to minimize thermal distortion

6.4 Materials for table legs

Luigi Tavolino, furniture designer, conceives of a lightweight table of daring simplicity: a flat sheet of toughened glass supported on slender, unbraced, cylindrical legs (Figure 6.5). The legs must be solid (to make them thin) and as light as possible (to make the table easier to move). They must support the table top and whatever is placed upon it without buckling. What materials could one recommend?

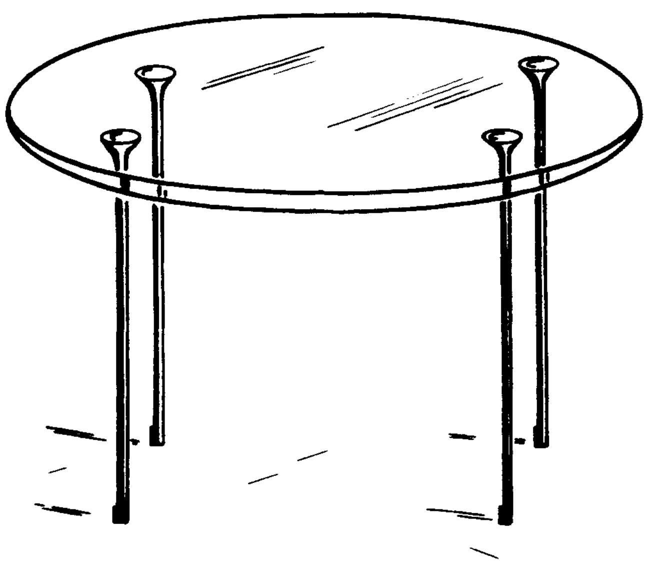

Fig. 6.5 A lightweight table with slender cylindrical legs. Lightness and slenderness are independent design goals, both constrained by the requirement that the legs must not buckle when the table is loaded. The best choice is a material with high values of both $E^{1/2}/\rho$ and E.

Table 6.5 Design requirements for table legs

Function	Column (supporting compressive loads)
Objective	(a) Minimize the mass (b) Maximize slenderness
Constraints	(a) Length ℓ specified (b) Must not buckle under design loads (c) Must not fracture if accidentally struck

The model

This is a problem with two objectives*: weight is to be minimized, and slenderness maximized. There is one constraint: resistance to buckling. Consider minimizing weight first.

The leg is a slender column of material of density ρ and modulus E. Its length, ℓ, and the maximum load, P, it must carry are determined by the design: they are fixed. The radius r of a leg is a free variable. We wish to minimize the mass m of the leg, given by the objective function

$$m = \pi r^2 \ell \rho \tag{6.6}$$

subject to the constraint that it supports a load P without buckling. The elastic load P_{crit} of a column of length ℓ and radius r (see Appendix A, 'Useful Solutions') is

$$P_{\text{crit}} = \frac{\pi^2 EI}{\ell^2} = \frac{\pi^3 E r^4}{4\ell^2} \tag{6.7}$$

using $I = \pi r^4/4$ where I is the second moment of area of the column. The load P must not exceed P_{crit}. Solving for the free variable, r, and substituting it into the equation for m gives

$$m \geq \left(\frac{4P}{\pi}\right)^{1/2} (\ell)^2 \left[\frac{\rho}{E^{1/2}}\right] \tag{6.8}$$

The material properties are grouped together in the last pair of brackets. The weight is minimized by selecting the subset of materials with the greatest value of the material index

$$\boxed{M_1 = \frac{E^{1/2}}{\rho}}$$

(a result we could have taken directly from Appendix B).

Now slenderness. Inverting equation (6.7) with $P = P_{\text{crit}}$ gives an equation for the thinnest leg which will not buckle:

$$r = \left(\frac{4P}{\pi^3}\right)^{1/4} (\ell)^{1/2} \left[\frac{1}{E}\right]^{1/4} \tag{6.9}$$

The thinnest leg is that made of the material with the largest value of the material index

$$\boxed{M_2 = E}$$

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* Formal methods for dealing with multiple objectives are developed in Chapter 9.

The selection

We seek the subset of materials which have high values of $E^{1/2}/\rho$ and E. Figure 6.6 shows the appropriate chart: Young's modulus, E, plotted against density, ρ. A guideline of slope 2 is drawn on the diagram; it defines the slope of the grid of lines for values of $E^{1/2}/\rho$. The guideline is displaced upwards (retaining the slope) until a reasonably small subset of materials is isolated above it; it is shown at the position $M_1 = 6\,\text{GPa}^{1/2}/(\text{Mg/m}^3)$. Materials above this line have higher values of

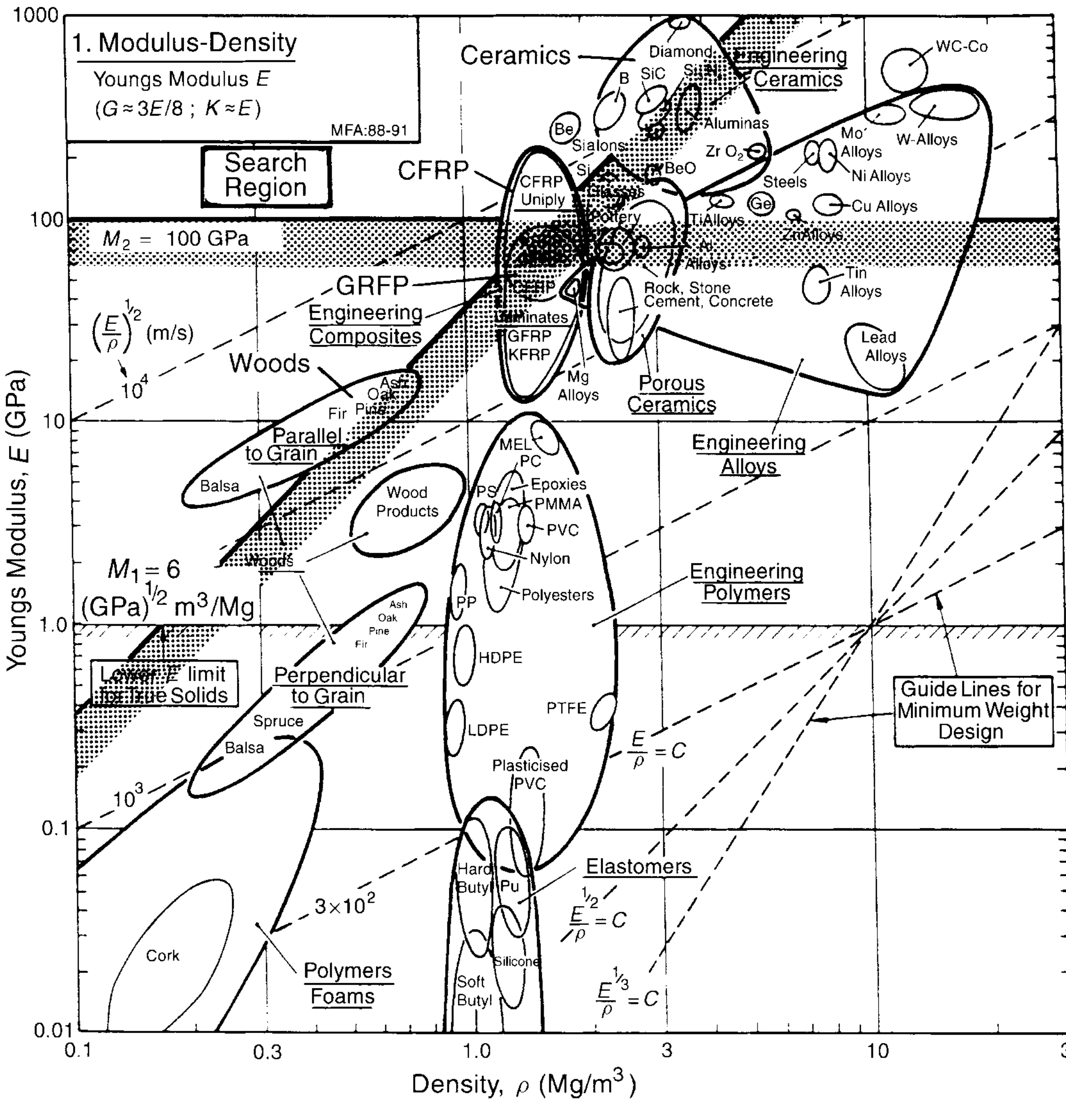

Fig. 6.6 Materials for light, slender legs. Wood is a good choice; so is a composite such as CFRP, which, having a higher modulus than wood, gives a column which is both light and slender. Ceramics meet the stated design goals, but are brittle.

Table 6.6 Materials for table legs

Material	M_1 $(GPa^{1/2}m^3/Mg)$	M_2 (GPa)	*Comment*
Woods	5–8	4–20	Outstanding M_1; poor M_2. Cheap, traditional, reliable.
CFRP	4–8	30–200	Outstanding M_1 and M_2, but expensive.
GFRP	3.5–5.5	20–90	Cheaper than CFRP, but lower M_1 and M_2.
Ceramics	4–8	150–1000	Outstanding M_1 and M_2. Eliminated by brittleness.

M_1. They are identified on the figure: *woods* (the traditional material for table legs), *composites* (particularly CFRP) and certain special *engineering ceramics*. Polymers are out: they are not stiff enough; metals too: they are too heavy (even magnesium alloys, which are the lightest). The choice is further narrowed by the requirement that, for slenderness, E must be large. A horizontal line on the diagram links materials with equal values of E; those above are stiffer. Figure 6.6 shows that placing this line at $M_1 = 100\,\text{GPa}$ eliminates woods and GFRP. If the legs must be really thin, then the shortlist is reduced to CFRP and ceramics: they give legs which weigh the same as the wooden ones but are much thinner. Ceramics, we know, are brittle: they have low values of fracture toughness. Table legs are exposed to abuse — they get knocked and kicked; common sense suggests that an additional constraint is needed, that of adequate toughness. This can be done using Chart 6 (Figure 4.7); it eliminates ceramics, leaving CFRP. The cost of CFRP (Chart 14, Figure 4.15) may cause Snr. Tavolino to reconsider his design, but that is another matter: he did not mention cost in his original specification.

It is a good idea to lay out the results as a table, showing not only the materials which are best, but those which are second-best — they may, when other considerations are involved become the best choice. Table 6.6 shows one way of doing it.

Postscript

Tubular legs, the reader will say, must be lighter than solid ones. True; but they will also be fatter. So it depends on the relative importance Mr Tavolino attaches to his two objectives — lightness and slenderness — and only he can decide that. If he can be persuaded to live with fat legs, tubing can be considered — and the material choice may be different. Materials selection when section-shape is a variable comes in Chapter 7.

Ceramic legs were eliminated because of low toughness. If (improbably) the goal was to design a light, slender-legged table for use at high temperatures, ceramics should be reconsidered. The brittleness problem can be by-passed by protecting the legs from abuse, or by pre-stressing them in compression.

Related case studies

Case Study 6.3: Mirrors for large telescopes
Case Study 8.2: Spars for man-powered planes
Case Study 8.3: Forks for a racing bicycle

6.5 Cost — structural materials for buildings

The most expensive thing that most people buy is the house they live in. Roughly half the cost of a house is the cost of the materials of which it is made, and they are used in large quantities (family house: around 200 tonnes; large apartment block: around 20 000 tonnes). The materials are used in three ways (Figure 6.7): structurally to hold the building up; as cladding, to keep the weather out; and as 'internals', to insulate against heat, sound, and so forth).

Consider the selection of materials for the structure. They must be stiff, strong, and cheap. Stiff, so that the building does not flex too much under wind loads or internal loading. Strong, so that there is no risk of it collapsing. And cheap, because such a lot of material is used. The structural frame of a building is rarely exposed to the environment, and is not, in general, visible. So criteria of corrosion resistance, or appearance , are not important here. The design goal is simple: strength and stiffness at minimum cost. To be more specific: consider the selection of material for floor beams. Table 6.7 summarizes the requirements.

The model

The way of deriving material indices for cheap, stiff and strong beams was developed in Chapter 5. The results we want are listed in Table 5.7. The critical components in building are loaded either

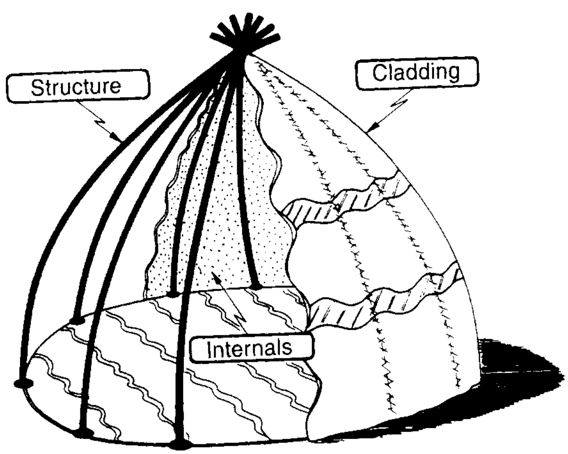

Fig. 6.7 The materials of a building perform three broad roles. The frame gives mechanical support; the cladding excludes the environment; and the internal surfacing controls heat, light and sound.

Table 6.7 Design requirements for floor beams

Function	Floor beams
Objective	Minimize the cost
Constraints	(a) Length L specified
	(b) Stiffness: must not deflect too much under design loads
	(c) Strength: must not fail under design loads

in bending (floor joists, for example) or as columns (the vertical members). The two indices that we want to maximize are:

$$M_1 = \frac{E^{1/2}}{\rho C_m}$$

and

$$M_2 = \frac{\sigma_f^{2/3}}{\rho C_m}$$

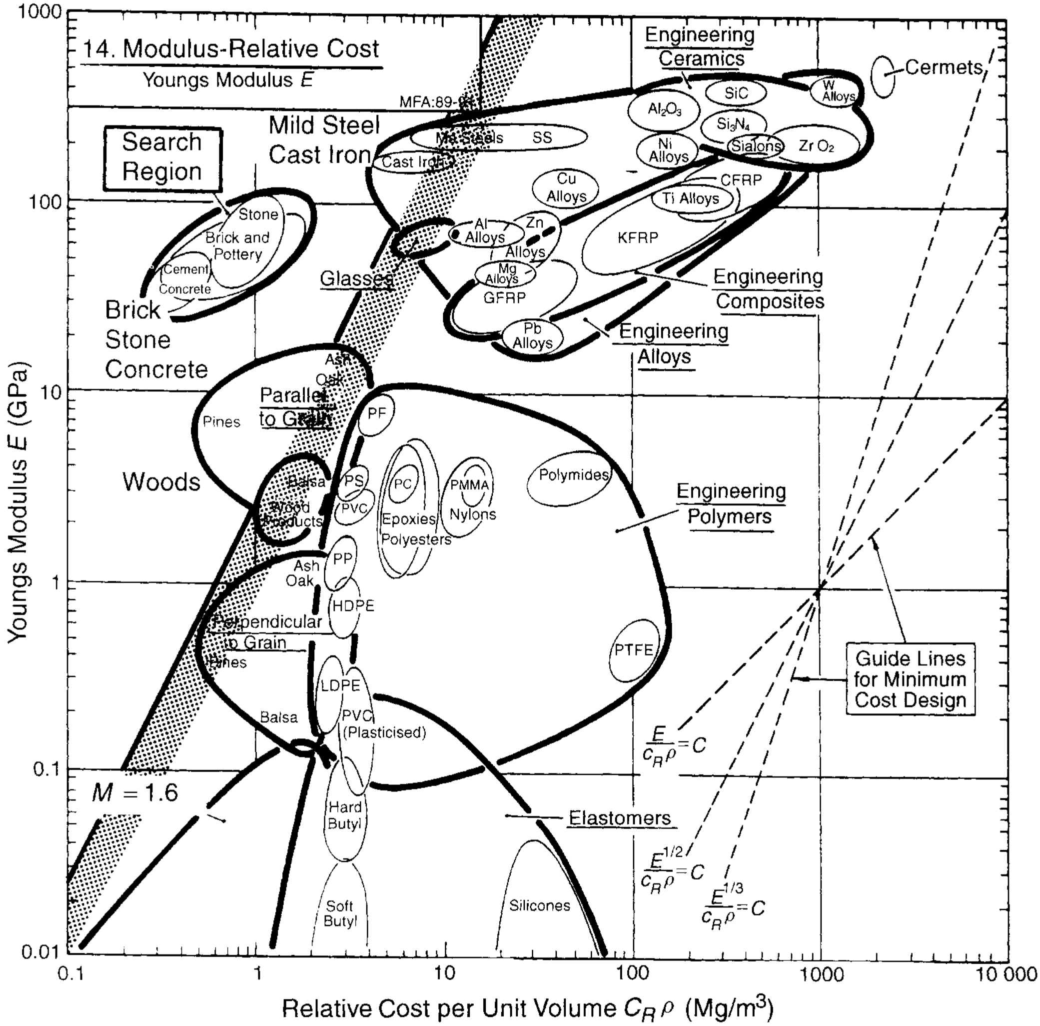

Fig. 6.8 The selection of cheap, stiff materials for the structural frames of buildings.

where, as always, E is Young's modulus, σ_f is the failure strength, ρ is the density and C_m material cost.

The selection

Cost appears in two of the charts. Figure 6.8 shows the first of them: modulus against relative cost per unit volume. The shaded band has the appropriate slope; it isolates concrete, stone, brick, softwoods, cast irons and the cheaper steels. The second, strength against relative cost, is shown in Figure 6.9. The shaded band — M_2 this time — gives almost the same selection. They are listed, with values, in the table. They are exactly the materials of which buildings have been, and are, made.

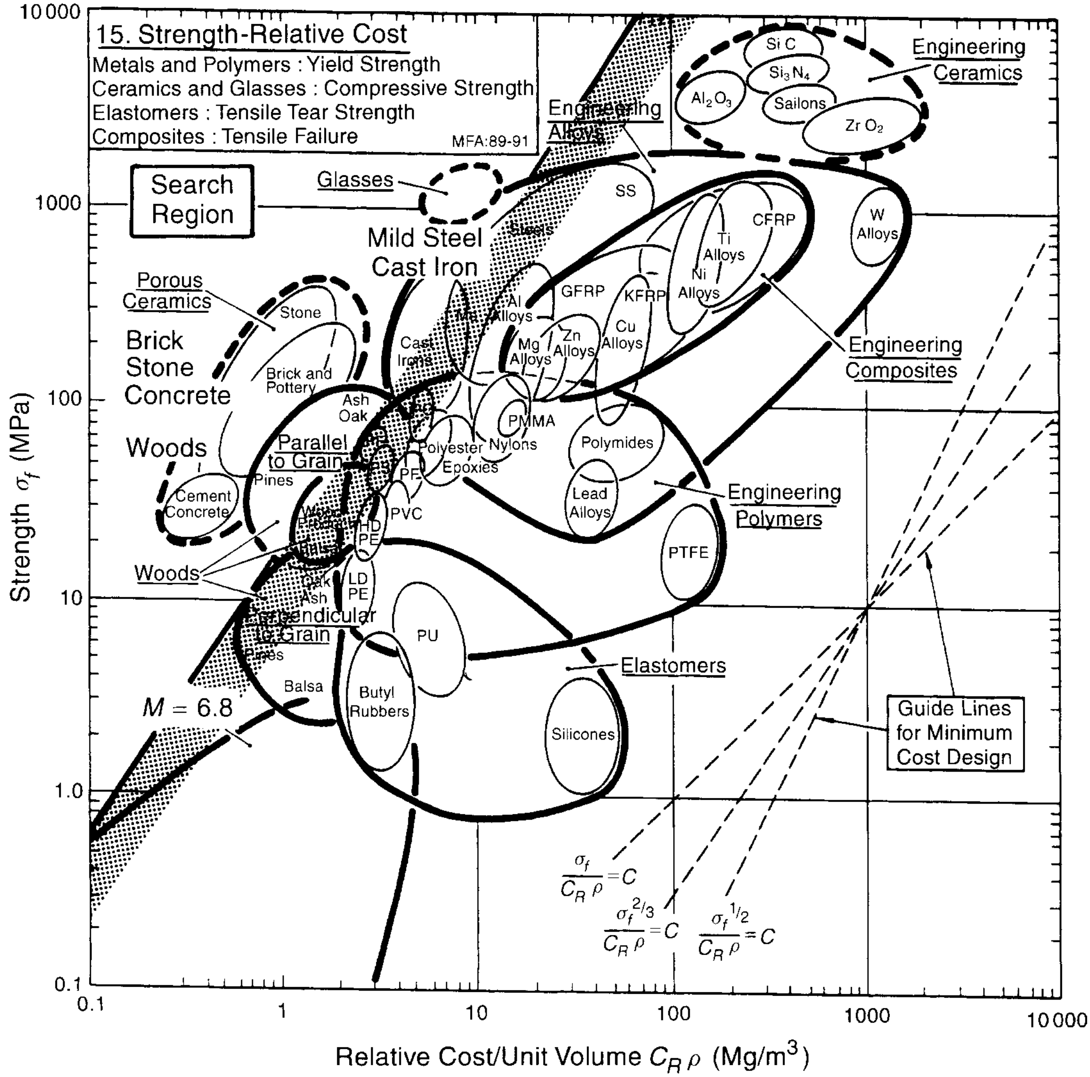

Fig. 6.9 The selection of cheap, strong materials for the structural frames of buildings.

Table 6.8 Structural materials for buildings

Material	M_1 $(GPa^{1/2}/(k\$/m^3))$	M_2 $(MPa^{2/3}/(k\$/m^3))$	*Comment*
Concrete	40	80	Use in compression only
Brick	20	45	
Stone	15	45	
Woods	15	80	Tension and compression, with freedom of section shape
Cast iron	5	20	
Steel	3	21	
Reinforced concrete	20	60	

Postscript

It is sometimes suggested that architects live in the past; that in the late 20th century they should be building with fibreglass (GFRP), aluminium alloys and stainless steel. Occasionally they do, but the last two figures give an idea of the penalty involved: the cost of achieving the same stiffness and strength is between 5 and 10 times greater. Civil construction (buildings, bridges, roads and the like) is materials-intensive: the cost of the material dominates the product cost, and the quantity used is enormous. Then only the cheapest of materials qualify, and the design must be adapted to use them. Concrete, stone and brick have strength only in compression; the form of the building must use them in this way (columns, arches). Wood, steel and reinforced concrete have strength both in tension and compression, and steel, additionally, can be given efficient shapes (I-sections, box sections, tubes); the form of the building made from these has much greater freedom.

Further reading

Cowan, H.J. and Smith, P.R. (1988) *The Science and Technology of Building Materials*, Van Nostrand-Reinhold, New York.

Related case studies

Case Study 6.2: Materials for oars
Case Study 6.4: Materials for table legs
Case Study 8.4: Floor joists: wood or steel?

6.6 Materials for flywheels

Flywheels store energy. Small ones — the sort found in children's toys — are made of lead. Old steam engines have flywheels; they are made of cast iron. More recently flywheels have been proposed for power storage and regenerative braking systems for vehicles; a few have been built, some of high-strength steel, some of composites. Lead, cast iron, steel, composites — there is a strange diversity here. What *is* the best choice of material for a flywheel?

An efficient flywheel stores as much *energy per unit weight* as possible, without *failing*. Failure (were it to occur) is caused by centrifugal loading: if the centrifugal stress exceeds the tensile strength (or fatigue strength) the flywheel flies apart. One constraint is that this should not occur.

The flywheel of a child's toy is not efficient in this sense. Its velocity is limited by the pulling-power of the child, and never remotely approaches the burst velocity. In this case, and for the flywheel of an automobile engine — we wish to maximize the *energy stored per unit volume* at a constant (specified) *angular velocity*. There is also a constraint on the outer radius, R, of the flywheel so that it will fit into a confined space.

The answer therefore depends on the application. The strategy for optimizing flywheels for efficient energy-storing systems differs from that for children's toys. The two alternative sets of design requirements are listed in Tables 6.9(a) and (b).

The model

An efficient flywheel of the first type stores as much energy per unit weight as possible, without failing. Think of it as a solid disc of radius R and thickness t, rotating with angular velocity ω (Figure 6.10). The energy U stored in the flywheel is

$$U = \frac{1}{2}J\omega^2 \tag{6.10}$$

Table 6.9(a) Design requirements for maximum-energy flywheel

Function	Flywheel for energy storage
Objective	Maximize kinetic energy per unit mass
Constraints	(a) Must not burst (b) Adequate toughness to give crack-tolerance

Table 6.9(b) Design requirements for limited-velocity flywheel

Function	Flywheel for child's toy
Objective	Maximize kinetic energy per unit volume
Constraints	Outer radius fixed

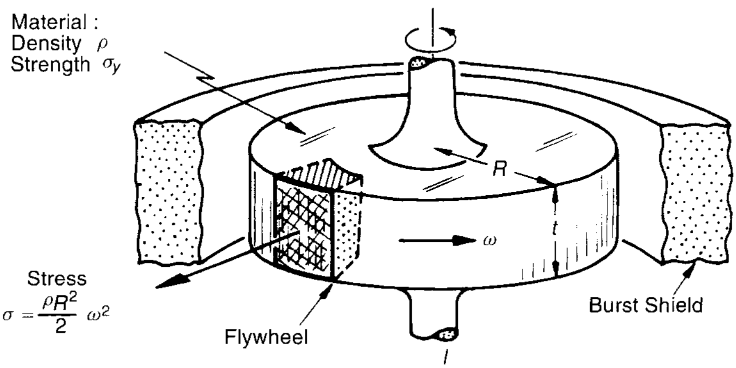

Fig. 6.10 A flywheel. The maximum kinetic energy it can store is limited by its strength.

Here $J = \frac{\pi}{2}\rho R^4 t$ is the polar moment of inertia of the disc and ρ the density of the material of which it is made, giving

$$U = \frac{\pi}{4}\rho R^4 t\omega^2 \qquad (6.11)$$

The mass of the disc is

$$m = \pi R^4 t\rho \qquad (6.12)$$

The quantity to be maximized is the kinetic energy per unit mass, which is the ratio of the last two equations:

$$\frac{U}{m} = \frac{1}{4}R^2\omega^2 \qquad (6.13)$$

As the flywheel is spun up, the energy stored in it increases, but so does the centrifugal stress. The maximum principal stress in a spinning disc of uniform thickness is

$$\sigma_{\max} = \left(\frac{3+\nu}{8}\right)\rho R^2\omega^2 \qquad (6.14)$$

where ν is Poisson's ratio. This stress must not exceed the failure stress σ_f (with an appropriate factor of safety, here omitted). This sets an upper limit to the angular velocity, ω, and disc radius, R (the free variables). Eliminating $R\omega$ between the last two equations gives

$$\frac{U}{m} = \left(\frac{2}{(3+\nu)}\right)\left(\frac{\sigma_f}{\rho}\right) \qquad (6.15)$$

Poissons's ratio, ν, is roughly 1/3 for solids; we can treat it as a constant. The best materials for high-performance flywheels are those with high values of the material index

$$\boxed{M = \frac{\sigma_f}{\rho}} \qquad (6.16)$$

It has units of kJ/kg.

But what of the other sort of flywheel — that of the child's toy? Here we seek the material which stores the most energy per unit volume V at constant velocity. The energy per unit volume at a given ω is (from equation (6.2)):

$$\frac{U}{V} = \frac{1}{4}\rho R^2\omega^2$$

Both R and ω are fixed by the design, so the best material is now that with the greatest value of

$$\boxed{M_2 = \rho} \qquad (6.17)$$

The selection

Figure 6.11 shows Chart 2: strength against density. Values of M correspond to a grid of lines of slope 1. One such line is shown at the value $M = 100\,\text{kJ/kg}$. Candidate materials with high values

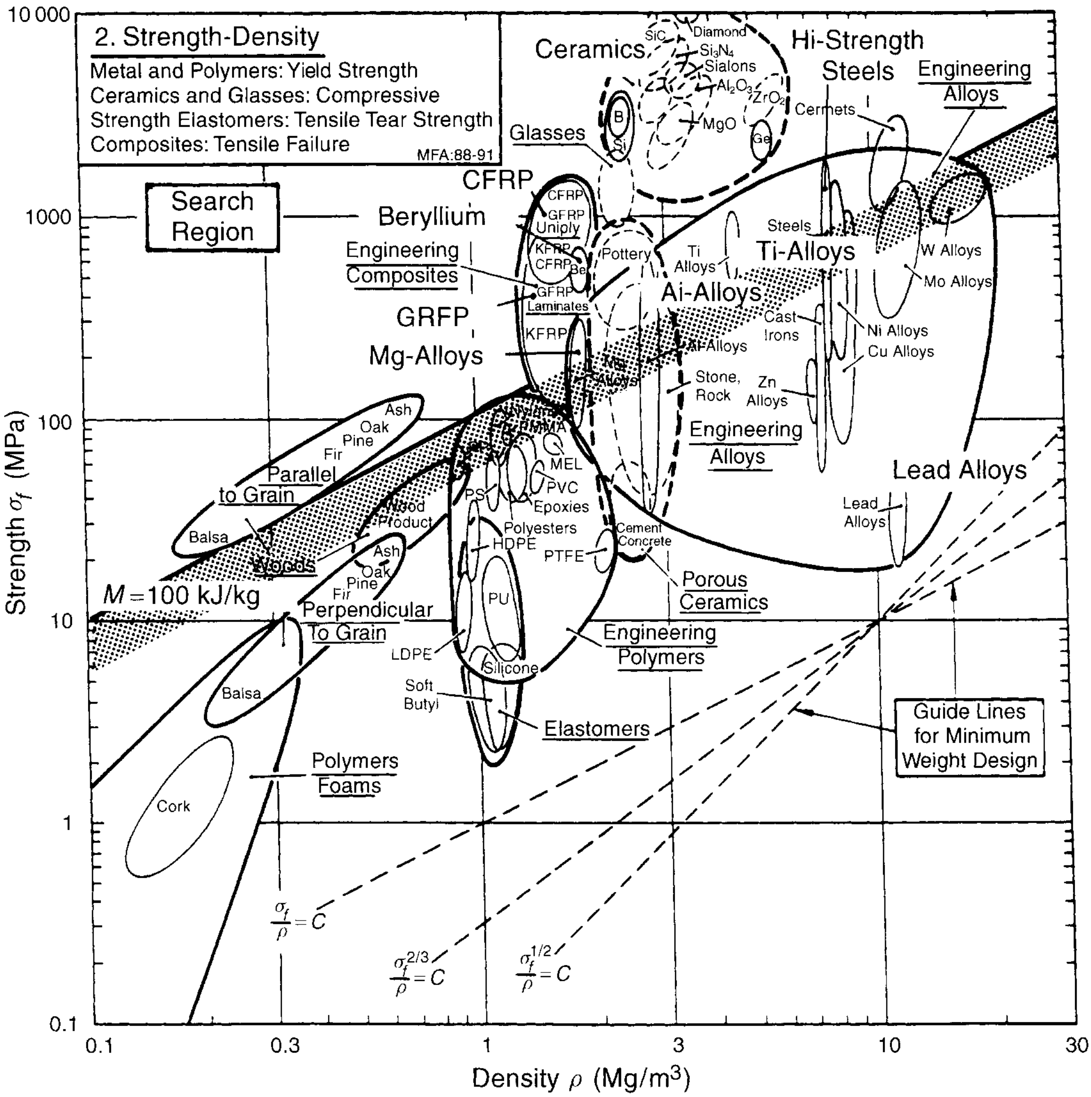

Fig. 6.11 Materials for flywheels. Composites and beryllium are the best choices. Lead and cast iron, traditional for flywheels, are good when performance is limited by rotational velocity, not strength.

of M lie in the search region towards the top left. They are listed in the upper part of Table 6.10. The best choices are unexpected ones: beryllium and composites, particularly glass-fibre reinforced polymers. Recent designs use a filament-wound glass-fibre reinforced rotor, able to store around 150 kJ/kg; a 20 kg rotor then stores 3 MJ or 800 kWh. A lead flywheel, by contrast, can store only 3 kJ/kg before disintegration; a cast-iron flywheel, about 10. All these are small compared with the energy density in gasoline: roughly 20 000 kJ/kg.

Even so, the energy density in the flywheel is considerable; its sudden release in a failure could be catastrophic. The disc must be surrounded by a burst-shield and precise quality control in manufacture is essential to avoid out-of-balance forces. This has been achieved in a number of

Table 6.10 Materials for flywheels

Material	*M (kJ/kg)*	*Comment*
Ceramics	200–2000 (compression only)	Brittle and weak in tension — eliminate.
Composites: CFRP	200–500	The best performance — a good choice.
GFRP	100–400	Almost as good as CFRP and cheaper. Excellent choice.
Beryllium	300	Good but expensive, difficult to work and toxic.
High-strength steel	100–200	All about equal
High-strength Al alloys	100–200	in performance. Steel and Al alloys
High-strength Mg alloys	100–200	cheaper than Mg and Ti alloys.
Ti alloys	100–200	
Lead alloys	3	High density makes these a good (and
Cast iron	8–10	traditional) selection when performance is velocity-limited, not strength-limited.

glass-fibre energy-storage flywheels intended for use in trucks and buses, and as an energy reservoir for smoothing wind-power generation.

But what of the lead flywheels of children's toys? There could hardly be two more different materials than GFRP and lead: the one, strong and light, the other, soft and heavy. Why lead? It is because, in the child's toy, the constraint is different. Even a super-child cannot spin the flywheel of his toy up to its burst velocity. The angular velocity ω is limited, instead, by the drive mechanism (pull-string, friction drive). Then, as we have seen, the best material is that with the largest density (Table 6.10, bottom section). Lead is good. Cast iron is less good, but cheaper. Gold, platinum and uranium are better, but may be thought unsuitable for other reasons.

Postscript

And now a digression: the electric car. By the turn of the century electric cars will be on the roads, powered by a souped-up version of the lead-acid battery. But batteries have their problems: the energy density they can contain is low (see Table 6.11); their weight limits both the range and the performance of the car. It is practical to build flywheels with an energy density of roughly five times that of the battery. Serious consideration is now being given to a flywheel for electric cars. A pair of counter-rotating CFRP discs are housed in a steel burst-shield. Magnets embedded in the discs pass near coils in the housing, inducing a current and allowing power to be drawn to the electric motor which drives the wheels. Such a flywheel could, it is estimated, give an electric car a range of 600 km, at a cost competitive with the gasoline engine.

Further reading

Christensen, R.M. (1979) *Mechanics of Composite Materials*, Wiley Interscience, New York, p. 213 *et seq.*
Lewis, G. (1990) *Selection of Engineering Materials*, Prentice Hall, Englewood Cliffs, NJ, Part 1, p. 1.
Medlicott, P.A.C. and Potter, K.D. (1986) The development of a composite flywheel for vehicle applications, in *High Tech — the Way into the Nineties*, edited by Brunsch, K., Golden, H-D., and Horkert, C-M. Elsevier, Amsterdam, p. 29.

Table 6.11 Energy density of power sources

Source	*Energy density kJ/kg*	*Comment*
Gasoline	20 000	Oxidation of hydrocarbon — mass of oxygen not included.
Rocket fuel	5000	Less than hydrocarbons because oxidizing agent forms part of fuel.
Flywheels	Up to 350	Attractive, but not yet proven.
Lead–acid battery	40–50	Large weight for acceptable range.
Springs rubber bands	Up to 5	Much less efficient method of energy storage than flywheel.

Related case studies

Case Study 6.7: Materials for high-flow fans
Case Study 6.15: Safe pressure vessels

6.7 Materials for high-flow fans

Automobile engines have a fan which cools the radiator when the forward motion of the car is insufficient to do the job. Commonly, the fan is driven by a belt from the main drive-shaft of the engine. The blades of the fan are subjected both to centrifugal forces and to bending moments caused by sudden acceleration of the motor. At least one fatality has been caused by the disintegration of a fan when an engine which had been reluctant to start suddenly sprang to life and was violently raced while a helper leaned over it. What criteria should one adopt in selecting materials to avoid this? The material chosen for the fan must be cheap. Any automaker who has survived to the present day has cut costs relentlessly on every component. But safety comes first.

The radius, R, of the fan is determined by design considerations: flow rate of air, and the space into which it must fit. The fan must not fail. The design requirements, then, are those of Table 6.12.

The model

A blade (Figure 6.12) has mean section area A and length αR, where α is the fraction of the fan radius R which is blade (the rest is hub). Its volume is αRA and the angular acceleration is $\omega^2 R$, so

Table 6.12 Design requirements for the fan

Function	Cooling fan
Objective	Maximum angular velocity without failure
Constraints	(a) Radius R specified (b) Must be cheap and easy to form

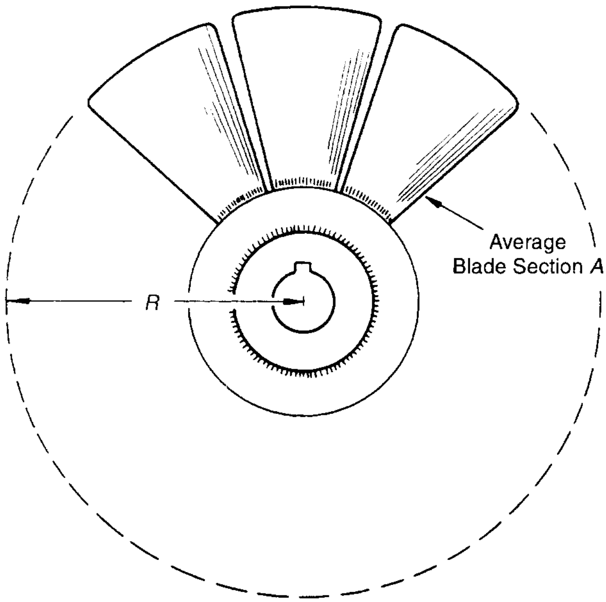

Fig. 6.12 A fan. The flow-rate of gas through the fan is related to its rotation speed, which is ultimately limited by its strength.

the centrifugal force at the blade root is

$$F = \rho(\alpha RA)\omega^2 R \tag{6.18}$$

The force is carried by the section A, so the stress at the root of the blade is

$$\sigma = \frac{F}{A} = \alpha\rho\omega^2 R^2 \tag{6.19}$$

This stress must not exceed the failure stress Σ_f divided by a safety factor (typically about 3) which does not affect the analysis and can be ignored. Thus for safety:

$$\omega < \frac{1}{\sqrt{\alpha}R}\left(\frac{\sigma_f}{\rho}\right)^{1/2}$$

The length R is fixed, as is α. The safe rotational velocity ω is maximized by selecting materials with large values of

$$\boxed{M = \frac{\sigma_f}{\rho}} \tag{6.21}$$

The selection

Figure 6.13 shows strength σ_f plotted against density, ρ. The materials above the selection line (slope = 1) have high values of M. This selection must be balanced against the cost. Low cost fans can be made by die-casting a metal, or by injection-moulding a polymer (Table 6.13).

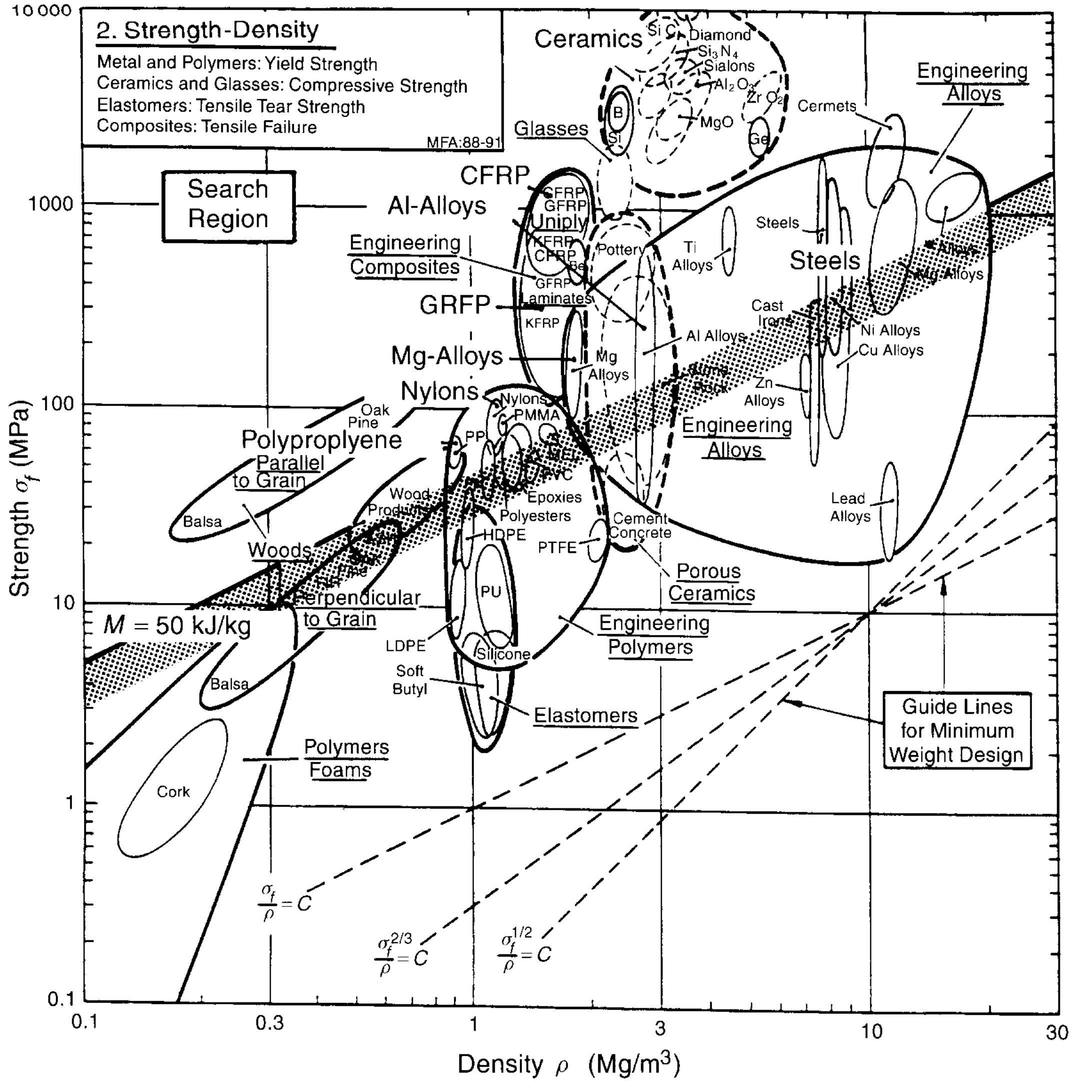

Fig. 6.13 Materials for cheap high-flow fans. Polymers — nylons and polypropylenes — are good; so are die-cast aluminium and magnesium alloys. Composites are better, but more difficult to fabricate.

Postscript

To an auto-maker additional cost is anathema, but the risk of a penal law suit is worse. Here (as elsewhere) it is possible to 'design' a way out of the problem. The problem is not really the fan; it is the undisciplined speed-changes of the engine which drives it. The solution (now we put it this way) is obvious: decouple the two. Increasingly, the cooling fans of automobiles are driven, not by the engine, but by an electric motor (cost: about that of a fan-belt) which limits it to speeds which are safe — and gives additional benefits in allowing independent control and more freedom in where the fan is placed.

Table 6.13 Candidate materials for a high-flow fan

Material	*Comment*
Cast iron	Cheap and easy to cast but poor σ_f/ρ.
Cast Al alloys	Can be die-cast to final shape.
High density polyethylene (HDPE) Nylons Rigid PVCs	Mouldable and cheap.
GFRP (chopped fibre) CFRP (chopped fibre)	Lay-up methods too expensive and slow. Press from chopped-fibre moulding material.

Related case studies

Case Study 6.6: Materials for flywheels
Case Study 12.2: Forming a fan
Case Study 14.3: A non-ferrous alloy: Al–Si die casting alloys

6.8 Golf-ball print heads

Mass is important when inertial forces are large, as they are in high-speed machinery. The golf-ball typewriter is an example: fast positioning of the golf-ball requires large accelerations and decelerations. Years before they came on the market, both the golf-ball and the daisy-wheel design had been considered and rejected: in those days print heads could only be made of heavy type-metal, and had too much inertia. The design became practical when it was realized that a polymer (density, 1 Mg/m^3) could be moulded to carry the type, replacing the lead-based type-metal (density, about 10 Mg/m^3). The same idea has contributed to other high-speed processes, which include printing, textile manufacture, and packaging.

The model

A golf-ball print head is a thin-walled shell with the type faces moulded on its outer surface (Figure 6.14). Its outer radius, R, is fixed by the requirement that it carry the usual 88 standard characters; the other requirements are summarized in Table 6.14. The time to reposition it varies as the square root of its mass, m, where

$$m \cong 4\pi R^2 t\rho \tag{6.22}$$

and t is the wall thickness and ρ the density of the material of which it is made. We wish to minimize this mass. The wall thickness must be sufficient to bear the strike force: a force F, distributed over

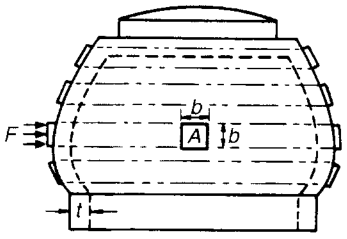

Fig. 6.14 A golf-ball print head. It must be strong yet light, to minimize inertial forces during rapid repositioning.

Table 6.14 Design requirements for golf-ball print heads

Function	Rapidly positioned print head
Objective	Minimize the mass (and thus inertia)
Constraints	(a) Outer radius R fixed (b) Adequate strength; must not fail under striking loads (c) Adequate stiffness (d) Can be moulded or cast to give sharply defined type-faces

an area of roughly b^2, where b is the average linear dimension of a character. When golf-ball print heads fail, they do so by cracking through the shell wall. We therefore require as a constraint that the through-thickness shear stress, $F/4bt$, be less than the failure strength, which, for shear, we approximate by $\sigma_f/2$:

$$\frac{F}{4bt} \leq \frac{\sigma_f}{2} \tag{6.23}$$

The free variable is the wall thickness, t. Solving for t and substituting into the equation (6.22) gives

$$m = F\left(\frac{2\pi R^2}{b}\right)\left(\frac{\rho}{\sigma_f}\right) \tag{6.24}$$

The repositioning time is minimized by choosing a material with the largest possible value of

$$\boxed{M = \frac{\sigma_f}{\rho}}$$

The material must also be mouldable or castable.

The selection

Materials for golf-balls require high σ_f/ρ; then Chart 2 is the appropriate one. It is reproduced in Figure 6.15, with appropriate selection lines constructed on it. It isolates two viable classes of candidate materials: metals, in the form of aluminium or magnesium casting alloys (which can be pressure die-cast) and the stronger polymers (which can be moulded to shape). Both classes, potentially, can meet the design requirements at a weight which is 15 to 20 times less than lead-based alloys which are traditional for type. We reject ceramics which are strong in compression but not in bending, and composites which cannot be moulded to give fine detail.

Data for the candidates are listed in Table 6.15, allowing a more detailed comparison. The final choice is an economic one: achieving high character-definition requires high-pressure moulding techniques which cost less, per unit, for polymers than for metals. High-modulus, high-strength polymers become the primary choice for the design.

Postscript

Printers are big business: long before computers were invented, IBM was already a large company made prosperous by selling typewriters. The scale of the market has led to sophisticated designs. Golf-balls and daisy-wheels are made of polymers, for the reasons given above; but not just one polymer. A modern daisy-wheel uses at least two: one for the type-face, which must resist wear

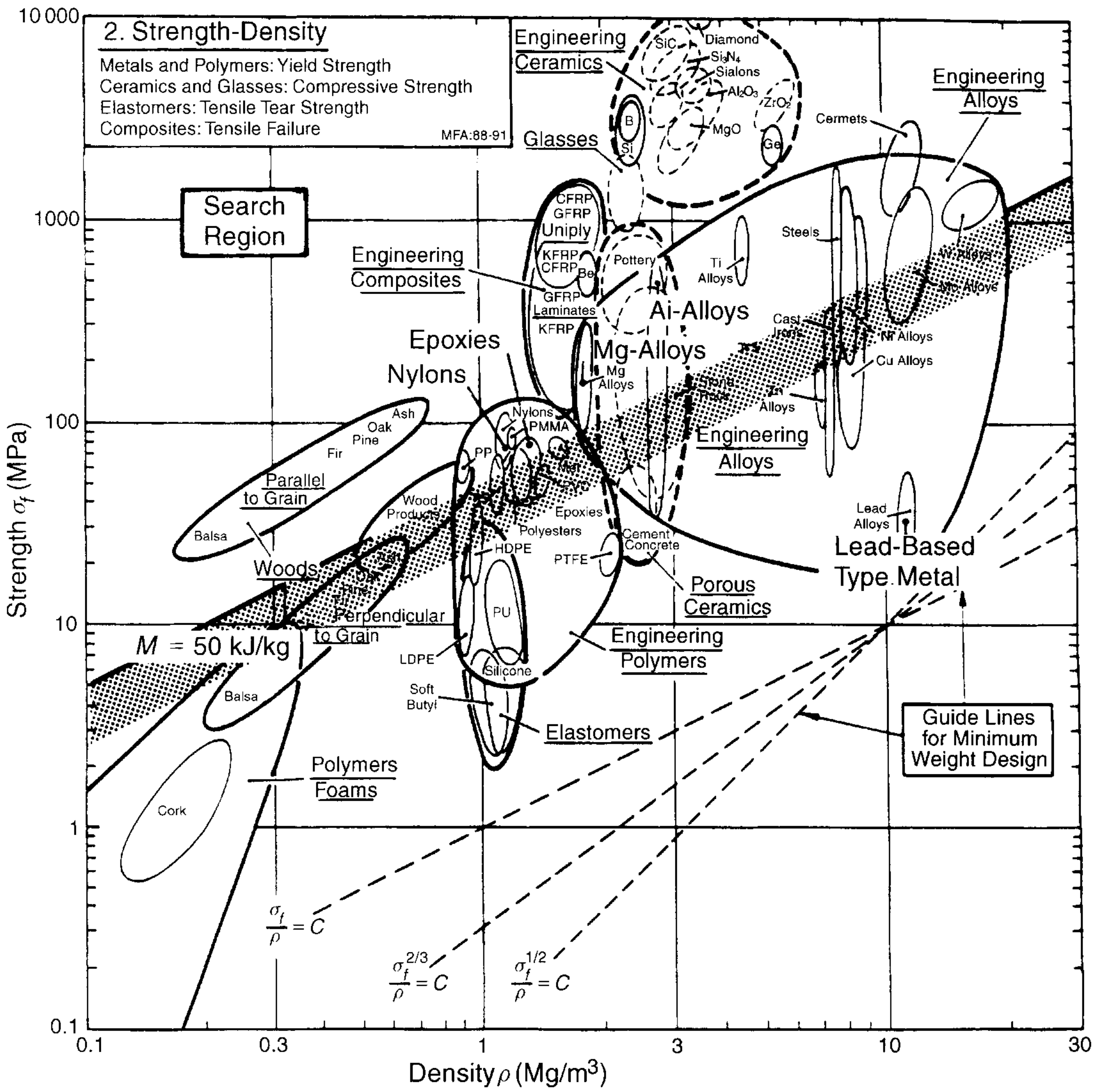

Fig. 6.15 Materials for golf-ball print heads. Polymers, because of their low density, are better than type-metal, which is mostly lead, and therefore has high inertia.

and impact, and a second for the fingers, which act as the return springs. Golf-balls have a surface coating for wear resistance, or simply to make the polymer look like a metal. Their days, however, are numbered. Laser and bubble-jet technologies have already largely displaced them. These, too, present problems in material selection, but of a different kind.

Related case studies

Case Study 6.6: Materials for flywheels
Case Study 6.7: Materials for high-flow fans

Table 6.15 Materials for golf-ball and daisy-wheel print heads

Material	$M = \frac{\sigma_f}{\rho}$ *(MPa/(Mg/m³))*	*Comment*
Nylons	80	Mouldable thermoplastic.
Epoxy	75	Castable thermoset.
Cast Mg alloys	60	Character definition poor.
Cast Al alloys	60	Character definition poor.
Type metal (Pb-5% Sn-10% Sb)	4	15 to 20 times heavier than the above for the same strength.

6.9 Materials for springs

Springs come in many shapes (Figure 6.16) and have many purposes: one thinks of axial springs (a rubber band, for example), leaf springs, helical springs, spiral springs, torsion bars. Regardless of their shape or use, the best material for a spring of minimum volume is that with the greatest value of σ_f^2/E, and for minimum weight it is that with the greatest value of $\sigma_f^2/E\rho$ (derived below). We use them as a way of introducing two of the most useful of the charts: Young's modulus E plotted against strength σ_f (Chart 4), and specific modulus, E/ρ, plotted against specific strength σ_f/ρ (Chart 5).

The model

The primary function of a spring is that of storing elastic energy and — when required — releasing it again (Table 6.16). The elastic energy stored per unit volume in a block of material stressed

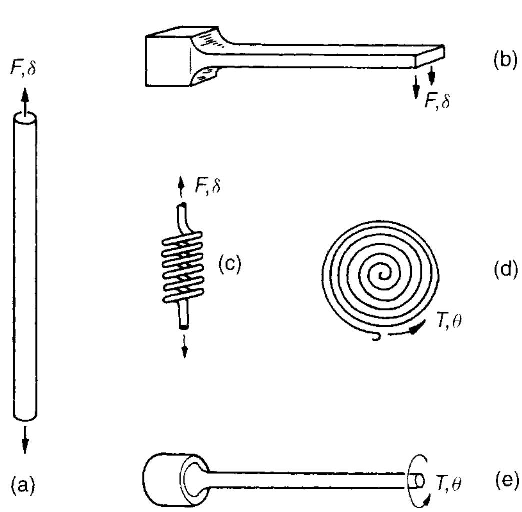

Fig. 6.16 Springs store energy. The best material for any spring, regardless of its shape or the way in which it is loaded, is that with the highest value of σ_f^2/E, or, if weight is important, $\sigma_f^2/E\rho$.

Table 6.16 Design requirements for springs

Function	Elastic spring
Objectives	(a) Maximum stored elastic energy per unit volume (b) Maximum stored elastic energy per unit mass
Constraints	(a) No failure by yield, fracture or fatigue (whichever is the most restrictive), meaning $\sigma < \sigma_f$ everywhere in the spring (b) Adequate toughness: $G_c > 1\ \mathrm{kJ/m^2}$

uniformly to a stress σ is

$$W_v = \frac{1}{2}\frac{\sigma^2}{E}$$

where E is Young's modulus. It is this W_v that we wish to maximize. The spring will be damaged if the stress σ exceeds the yield stress or failure stress σ_f; the constraint is $\sigma \leq \sigma_f$. So the maximum energy density is

$$W_v = \frac{1}{2}\frac{\sigma_f^2}{E} \tag{6.25}$$

Torsion bars and leaf springs are less efficient than axial springs because much of the material is not fully loaded: the material at the neutral axis, for instance, is not loaded at all. For torsion bars

$$W_v = \frac{1}{3}\frac{\sigma_f^2}{E}$$

and for leaf springs

$$W_v = \frac{1}{4}\frac{\sigma_f^2}{E}$$

But — as these results show — this has no influence on the choice of material. The best material for springs is that with the biggest value of

$$\boxed{M_1 = \frac{\sigma_f^2}{E}} \tag{6.26}$$

If weight, rather than volume, matters, we must divide this by the density ρ (giving energy stored per unit weight), and seek materials with high values of

$$\boxed{M_2 = \frac{\sigma_f^2}{\rho E}} \tag{6.27}$$

The selection

The choice of materials for springs of minimum volume is shown in Figure 6.17. A family lines of slope 1/2 link materials with equal values of $M_1 = \sigma_f^2/E$; those with the highest values of M_1

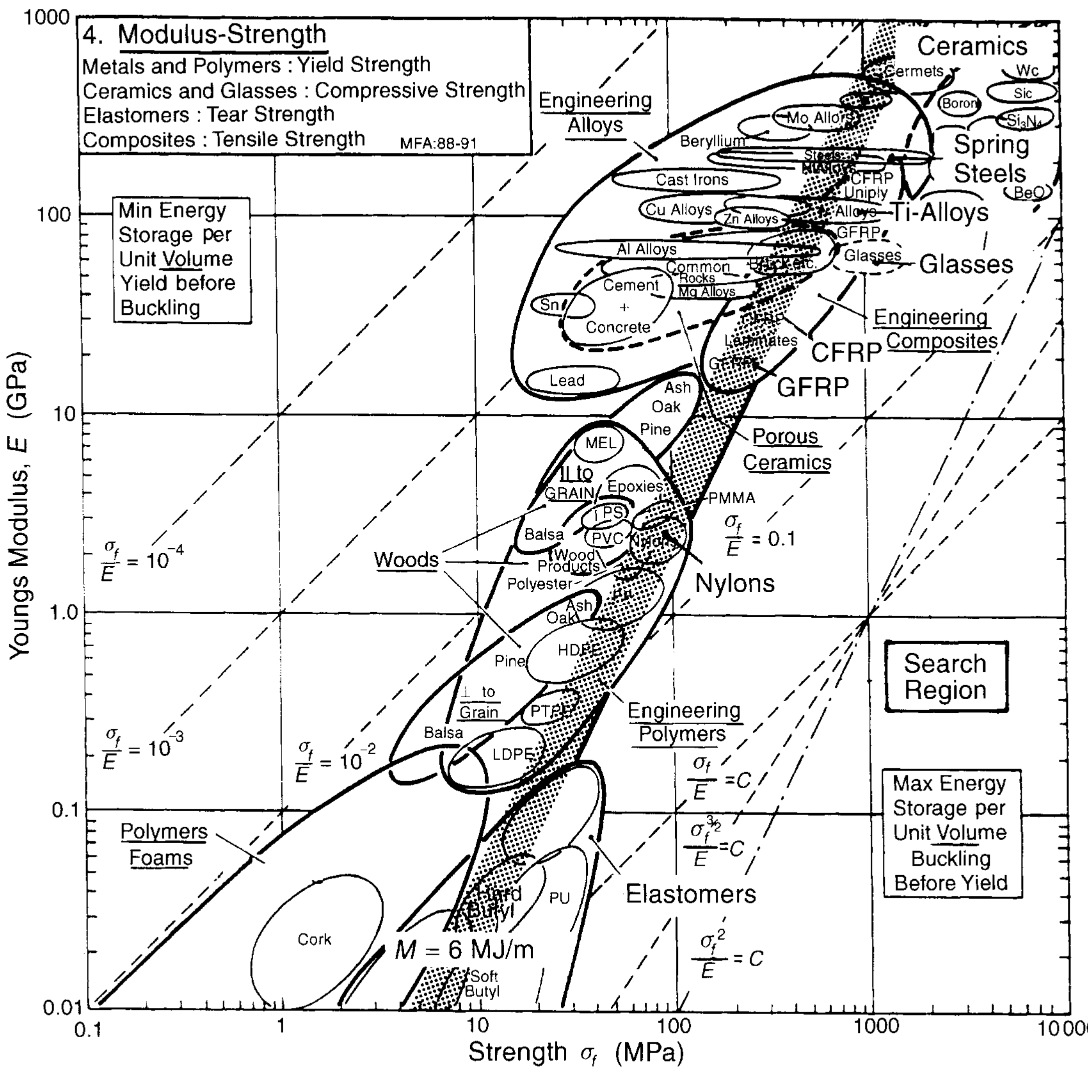

Fig. 6.17 Materials for small springs. High strength ('spring') steel is good. Glass, CFRP and GFRP all, under the right circumstances, make good springs. Elastomers are excellent. Ceramics are eliminated by their low tensile strength.

lie towards the bottom right. The heavy line is one of the family; it is positioned so that a subset of materials is left exposed. The best choices are a *high-strength steel* ((spring steel, in fact) lying near the top end of the line, and, at the other end, *rubber*. But certain other materials are suggested too: *GFRP* (now used for truck springs), *titanium alloys* (good but expensive), *glass* (used in galvanometers) and *nylon* (children's toys often have nylon springs). Note how the procedure has identified a candidate from almost every class of material: metals, glasses, polymers, elastomers and composites. They are listed, with commentary, in Table 6.17.

Table 6.17 Materials for efficient small springs

Material	$M_1 = \frac{\sigma_f^2}{E}$ *(MJ/m³)*	*Comment*
Ceramics	(10–100)	Brittle in tension; good only in compression.
Spring steel	15–25	The traditional choice: easily formed and heat treated.
Ti alloys	15–20	Expensive, corrosion-resistant.
CFRP	15–20	Comparable in performance with steel; expensive.
GFRP	10–12	Almost as good as CFRP and much cheaper.
Glass (fibres)	30–60	Brittle in torsion, but excellent if protected against damage; very low loss factor.
Nylon	1.5–2.5	The least good; but cheap and easily shaped, but high loss factor.
Rubber	20–50	Better than spring steel; but high loss factor.

Materials selection for light springs is shown in Figure 6.18. A family of lines of slope 2 link materials with equal values of

$$M_2 = \left(\frac{\sigma_f}{\rho}\right)^2 \bigg/ \left(\frac{E}{\rho}\right) = \frac{\sigma_f^2}{E\rho}$$

One is shown at the value $M_2 = 2\,\text{kJ/kg}$. Metals, because of their high density, are less good than composites, and much less good than elastomers. (You can store roughly eight times more elastic energy, per unit weight, in a rubber band than in the best spring steel.) Candidates are listed in Table 6.18. Wood, the traditional material for archery bows, now appears.

Postscript

Many additional considerations enter the choice of a material for a spring. Springs for vehicle suspensions must resist fatigue and corrosion; IC valve springs must cope with elevated temperatures. A subtler property is the loss coefficient, shown in Chart 7. Polymers have a relatively high loss factor and dissipate energy when they vibrate; metals, if strongly hardened, do not. Polymers, because they creep, are unsuitable for springs which carry a steady load, though they are still perfectly good for catches and locating-springs which spend most of their time unstressed.

Further reading

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Related case studies

Case Study 6.10: Elastic hinges
Case Study 6.12: Diaphragms for pressure actuators
Case Study 8.6: Ultra-efficient springs

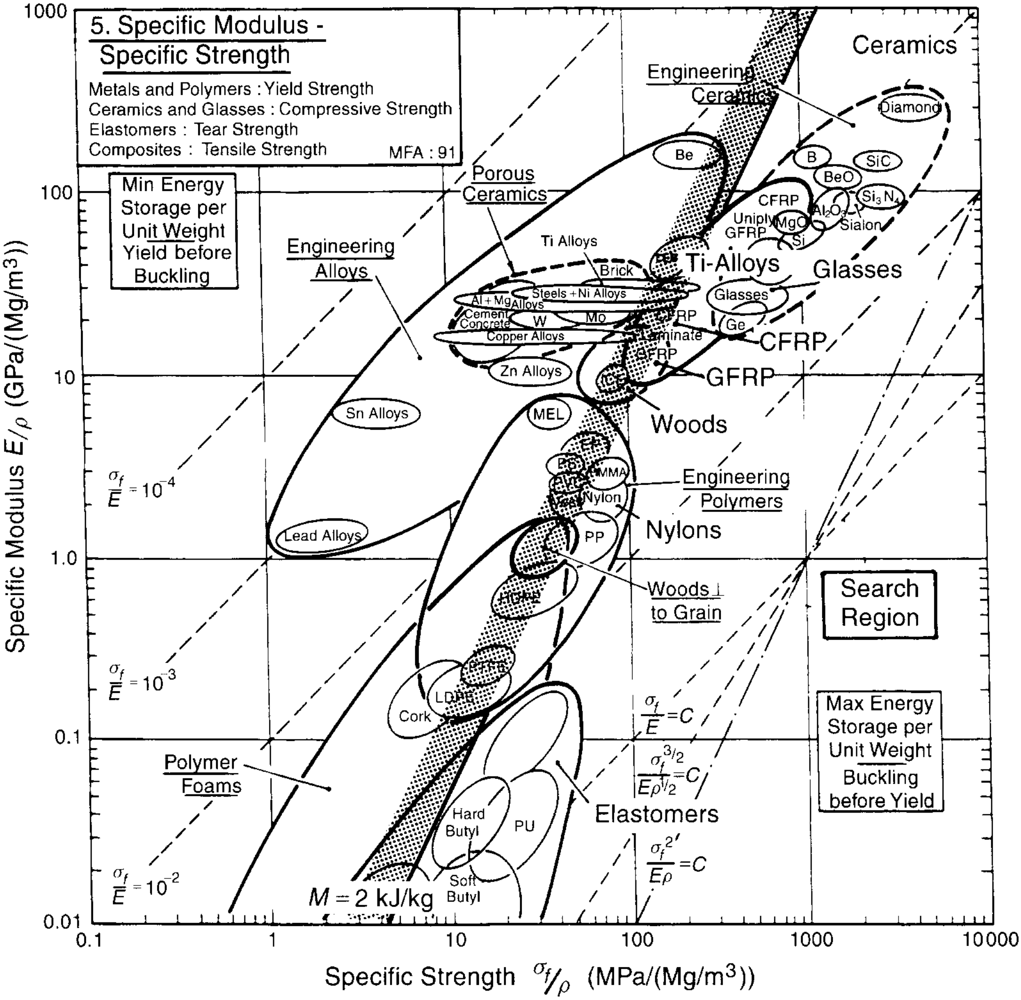

Fig. 6.18 Materials for light springs. Metals are disadvantaged by their high densities. Composites are good; so is wood. Elastomers are excellent.

Table 6.18 Materials for efficient light springs

Material	$M_2 = \dfrac{\sigma_f^2}{E\rho}$ *(kJ/kg)*	*Comment*
Ceramics	(5–40)	Brittle in tension; good only in compression.
Spring steel	2–3	Poor, because of high density.
Ti alloys	2–3	Better than steel; corrosion-resistant; expensive.
CFRP	4–8	Better than steel; expensive.
GFRP	3–5	Better than steel; less expensive than CFRP.
Glass (fibres)	10–30	Brittle in torsion, but excellent if protected.
Wood	1–2	On a weight basis, wood makes good springs.
Nylon	1.5–2	As good as steel, but with a high loss factor.
Rubber	20–50	Outstanding; 10 times better than steel, but with high loss factor.

6.10 Elastic hinges

Nature makes much use of elastic hinges: skin, muscle, cartilage all allow large, recoverable deflections. Man, too, designs with *flexure and torsion hinges*: devices which connect or transmit load between components while allowing limited relative movement between them by deflecting elastically (Figure 6.19 and Table 6.19). Which materials make good hinges?

The model

Consider the hinge for the lid of a box. The box, lid and hinge are to be moulded in one operation. The hinge is a thin ligament of material which flexes elastically as the box is closed, as in the figure, but it carries no significant axial loads. Then the best material is the one which (for given ligament dimensions) bends to the smallest radius without yielding or failing. When a ligament of thickness t is bent elastically to a radius R, the surface strain is

$$\varepsilon = \frac{t}{2R} \tag{6.28}$$

and, since the hinge is elastic, the maximum stress is

$$\sigma \geq E\frac{t}{2R} \tag{6.29}$$

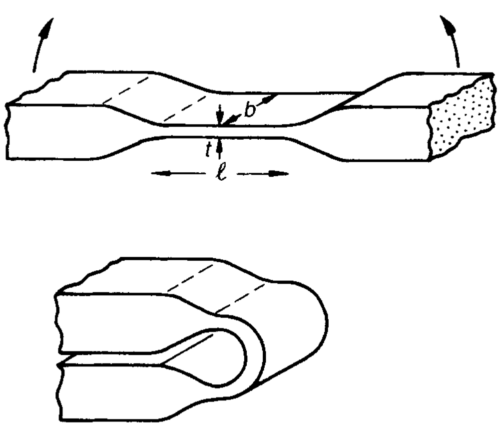

Fig. 6.19 Elastic or 'flexure' hinges. The ligaments must bend repeatedly without failing. The cap of a shampoo bottle is an example; elastic hinges are used in high performance applications too, and are found widely in nature.

Table 6.19 Design requirements for elastic hinges

Function	Elastic hinge (possibly with additional axial load)
Objective	Maximize elastic flexure or twisting
Constraints	No failure by yield, fracture or fatigue (whichever is the most restrictive) (a) with no axial load (b) with additional axial load

This must not exceed the yield or failure strength σ_f. Thus the radius to which the ligament can be bent without damage is

$$R \leq \frac{t}{2}\left[\frac{E}{\sigma_f}\right] \tag{6.30}$$

The best material is the one that can be bent to the smallest radius, that is, the one with the greatest value of the index

$$\boxed{M_1 = \frac{\sigma_f}{E}}$$

We have assumed thus far that the hinge thickness, t, is dictated by the way the hinge is made. But in normal use, the hinge may also carry repeated axial (tensile) forces, F, due to handling or to the weight of the box and its contents. This sets a minimum value for the thickness, t, which is found by requiring that the tensile stress, F/tw (where w is the hinge width) does not exceed the strength limit σ_f:

$$t^* = \frac{F}{\sigma_f w}$$

Substituting this value of t into equation (6.30) gives

$$R \leq \frac{F}{2w}\left[\frac{E}{\sigma_f^2}\right]$$

and the second index

$$\boxed{M_2 = \frac{\sigma_f^2}{E}}$$

The selection

The criteria both involve ratios of σ_f and E; we need Chart 4 (Figure 6.20). Candidates are identified by using the guide line of slope 1; a line is shown at the position $M = \sigma_y/E = 3 \times 10^{-2}$. The best choices for the hinge are all polymeric materials. The shortlist (Table 6.20) includes polyethylenes, polypropylene, nylon and, best of all, elastomers, though these may be too flexible for the body of the box itself. Cheap products with this sort of elastic hinge are generally moulded from polyethylene, polypropylene or nylon. Spring steel and other metallic spring materials (like phosphor bronze) are possibilities: they combine usable σ_f/E with high E, giving flexibility with good positional stability (as in the suspensions of relays). The tables gives further details.

Postscript

Polymers give more design-freedom than metals. The elastic hinge is one example of this, reducing the box, hinge and lid (three components plus the fasteners needed to join them) to a single box-hinge-lid, moulded in one operation. Their spring-like properties allow snap-together, easily-joined

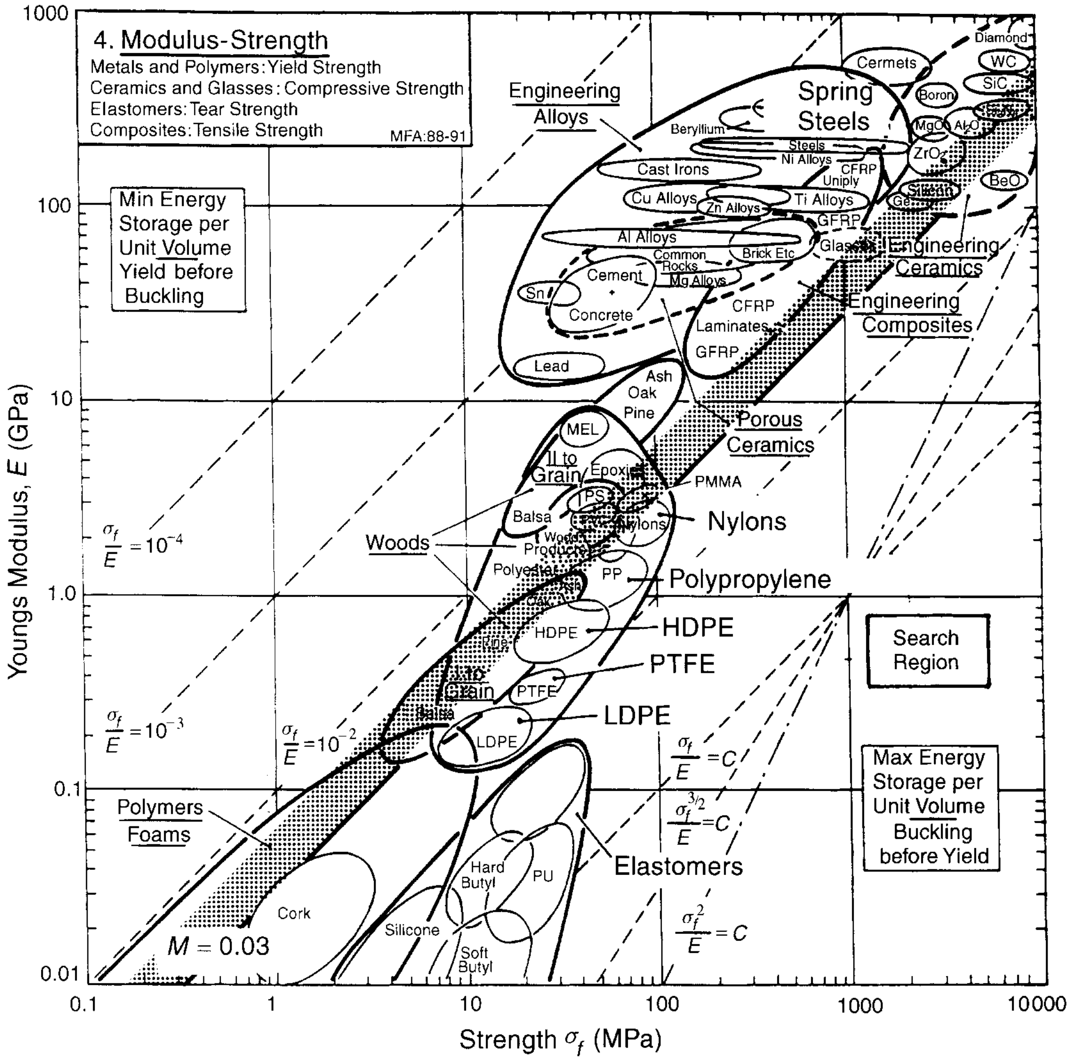

Fig. 6.20 Materials for elastic hinges. Elastomers are best, but may not be rigid enough to meet other design needs. Then polymers such as nylon, PTFE and PE are better. Spring steel is less good, but much stronger.

parts. Another is the elastomeric coupling — a flexible universal joint, allowing an exceptionally high angular, parallel and axial flexibility with good shock absorption characteristics. Elastomeric hinges offer many more opportunities, to be exploited in engineering design.

Related case studies

Case Study 6.9: Materials for springs
Case Study 6.11: Materials for seals
Case Study 6.12: Diaphragms for pressure actuators

Table 6.20 Materials for elastic hinges

Material	M_1 $(\times 10^{-3})$	M_2 (MJ/m^3)	*Comment*
Polyethylenes	30–45	1.6–1.8	Widely used for cheap hinged bottle caps, etc.
Polypropylene	30	1.6–1.7	Stiffer than PEs. Easily moulded.
Nylon	30	2–2.1	Stiffer than PEs. Easily moulded.
PTFE	35	2–2.1	Very durable; more expensive than PE, PP, etc.
Elastomers	100–300	10–20	Outstanding, but low modulus.
Beryllium-copper	5–10	8–12	M_1 less good than polymers. Use when high stiffness required.
Spring steel	5–10	10–20	M_1 less good than polymers. Use when high stiffness required.

6.11 Materials for seals

A reusable elastic seal consists of a cylinder of material compressed between two flat surfaces (Figure 6.21). The seal must form the largest possible contact width, b, while keeping the contact stress, σ sufficiently low that it does not damage the flat surfaces; and the seal itself must remain elastic so that it can be reused many times. What materials make good seals? Elastomers — everyone knows that. But let us do the job properly; there may be more to be learnt. We build the selection around the requirements of Table 6.21.

The model

A cylinder of diameter $2R$ and modulus E, pressed on to a rigid flat surface by a force f per unit length, forms an elastic contact of width b (Appendix A: 'Useful Solutions') where

$$b \approx 2\left(\frac{fR}{E}\right)^{1/2} \tag{6.31}$$

This is the quantity to be maximized: the objective function. The contact stress, both in the seal and in the surface, is adequately approximated (Appendix A again) by

$$\sigma = 0.6\left(\frac{fE}{R}\right)^{1/2} \tag{6.32}$$

The constraint: the seal must remain elastic, that is, σ must be less than the yield or failure strength, σ_f, of the material of which it is made. Combining the last two equations with this condition gives

$$b \leq 3.3R\left(\frac{\sigma_f}{E}\right) \tag{6.33}$$

The contact width is maximized by maximizing the index

$$\boxed{M_1 = \frac{\sigma_f}{E}}$$

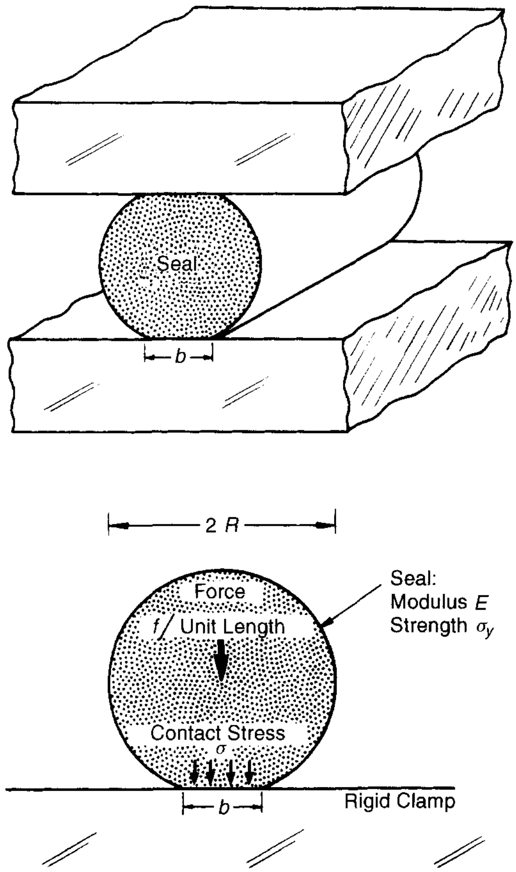

Fig. 6.21 An elastic seal. A good seal gives a large conforming contact area without imposing damaging loads on itself or on the surfaces with which it mates.

Table 6.21 Design requirements for the elastic seals

Function	Elastic seal
Objective	Maximum conformability
Constraints	(a) Limit on contact pressure (b) low cost

It is also required that the contact stress σ be kept low to avoid damage to the flat surfaces. Its value when the maximum contact force is applied (to give the biggest width) is simply σ_f, the failure strength of the seal. Suppose the flat surfaces are damaged by a stress of greater than 100 MPa. The contact pressure is kept below this by requiring that

$$M_2 = \sigma_f \leq 100\,\text{MPa}$$

The selection

The two indices are plotted on the $\sigma_f - E$ chart in Figure 6.22 isolating elastomers, foams and cork. The candidates are listed in Table 6.22 with commentary. The value of $M_2 = 100\,\text{MPa}$ admits all elastomers as candidates. If M_2 were reduced to 10 MPa, all but the most compliant elastomers are eliminated, and foamed polymers become the best bet.

Postscript

The analysis highlights the functions that seals must perform: large contact area, limited contact pressure, environmental stability. Elastomers maximize the contact area; foams and cork minimize

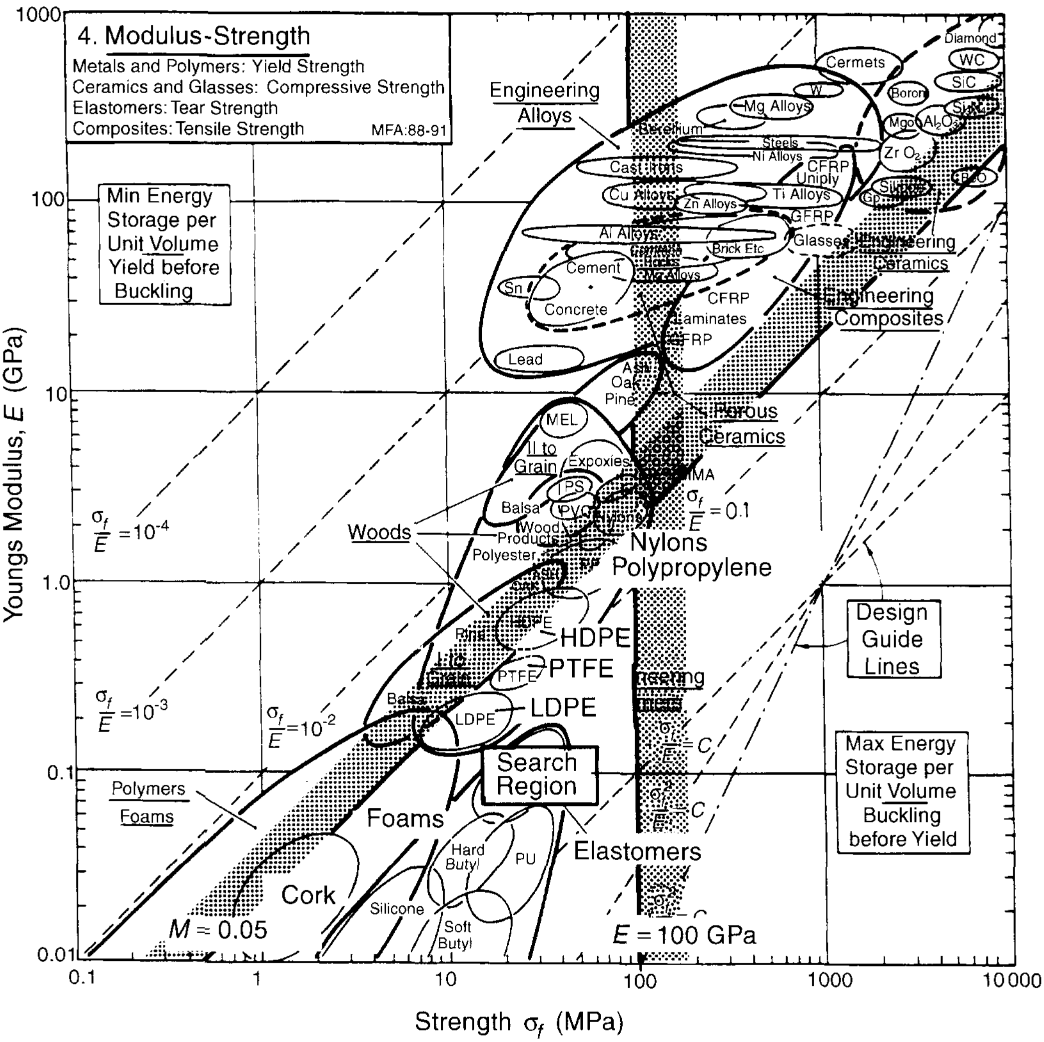

Fig. 6.22 Materials for elastic seals. Elastomers, compliant polymers and foams make good seals.

Table 6.22 Materials for reusable seals

Material	$M_1 = \frac{\sigma_f}{E}$	*Comment*
Butyl rubbers	1–3	The natural choice; poor resistance to heat and to some solvents.
Polyurethanes	0.5–4.5	Widely used for seals.
Silicone rubbers	0.1–0.8	Higher temperature capability than carbon-chain elastomers, chemically inert.
PTFE	0.1	Expensive but chemically stable and with high temperature capability.
Polyethylenes	0.05–0.2	Cheap.
Polypropylenes	0.1	Cheap.
Nylons	0.05	Near upper limit on contact pressure.
Cork	0.1	Low contact stress, chemically stable.
Polymer foams	up to 0.5	Very low contact pressure; delicate seals.

the contact pressure; PTFE and silicone rubbers best resist heat and organic solvents. The final choice depends on the conditions under which the seal will be used.

Related case studies

Case Study 6.9: Materials for springs
Case Study 6.10: Elastic hinges
Case Study 6.12: Diaphragms for pressure actuators
Case Study 6.13: Knife edges and pivots

6.12 Diaphragms for pressure actuators

A barometer is a pressure actuator. Changes in atmospheric pressure, acting on one side of a diaphragm, cause it to deflect; the deflection is transmitted through mechanical linkage or electromagnetic sensor to a read-out. Similar diaphragms form the active component of altimeters, pressure gauges, and gas-flow controls for diving equipment. Which materials best meet the requirements for diaphragms, summarized in Table 6.23?

The model

Figure 6.23 shows a diaphragm of radius a and thickness t. A pressure difference $\Delta p = p_1 - p_2$ acts across it. We wish to maximize the deflection of the centre of the diaphragm, subject to the

Table 6.23 Design requirements for diaphragms

Function	Diaphragm for pressure sensing
Objective	Maximize displacement for given pressure difference
Constraints	(a) Must remain elastic (no yield or fracture) (b) No creep (c) Low damping for quick, accurate response

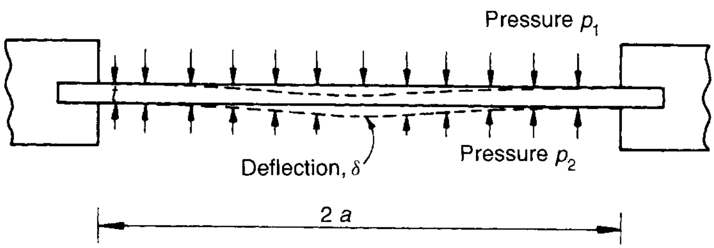

Fig. 6.23 A diaphragm. Its deflection under a pressure difference is used to sense and actuate.

constraint that it remain elastic — that is, that the stresses in it are everywhere less than the yield or fracture stress, σ_f, of the material of which it is made. The deflection δ of a diaphragm caused by Δp (Appendix A: 'Useful Solutions') depends on whether its edges are clamped or free:

$$\delta = \frac{C_1 \Delta p a^4 (1 - \nu^2)}{E t^3} \tag{6.34}$$

with $C_1 = \frac{3}{16}$ (clamped edges)

or $C_1 \approx \frac{9}{8}$ (free edges)

Here E is Young's modulus, and ν is Poisson's ratio. The maximum stress in the diaphragm (Appendix A again) is

$$\sigma_{\max} = C_2 \Delta p \frac{a^2}{t^2} \tag{6.35}$$

with $C_1 \approx \frac{1}{2}$ (clamped edges)

or $C_2 \approx \frac{3}{2}$ (free edges)

This stress must not exceed the yield or failure stress, σ_f.

The radius of the diaphragm is determined by the design; the thickness t is free. Eliminating t between the two equations gives

$$\delta = \frac{C_1}{C_2^{3/2}} \left(\frac{a}{\Delta p^{1/2}} \right) \left(\frac{\sigma_f^{3/2}(1 - \nu^2)}{E} \right) \tag{6.36}$$

The material properties are grouped in the last brackets. The quantity $(1 - \nu^2)$ is close to 1 for all solids. The best material for the diaphragm is that with the largest value of

$$\boxed{M = \frac{\sigma_f^{3/2}}{E}} \tag{6.37}$$

The selection

Figure 6.24 shows the selection. Candidates with large values of M are listed in Table 6.24 together with approximate values of their loss coefficients, η read from Chart 8. Ceramics are eliminated because the stresses of equation (6.35) are tensile. Metals make good diaphragms, notably spring steel, and high-strength titanium alloys. Certain polymers are possible — nylon, polypropylene and PTFE — but they have high damping and they creep. So do elastomers: both natural and artificial rubbers acquire a permanent set under static loads.

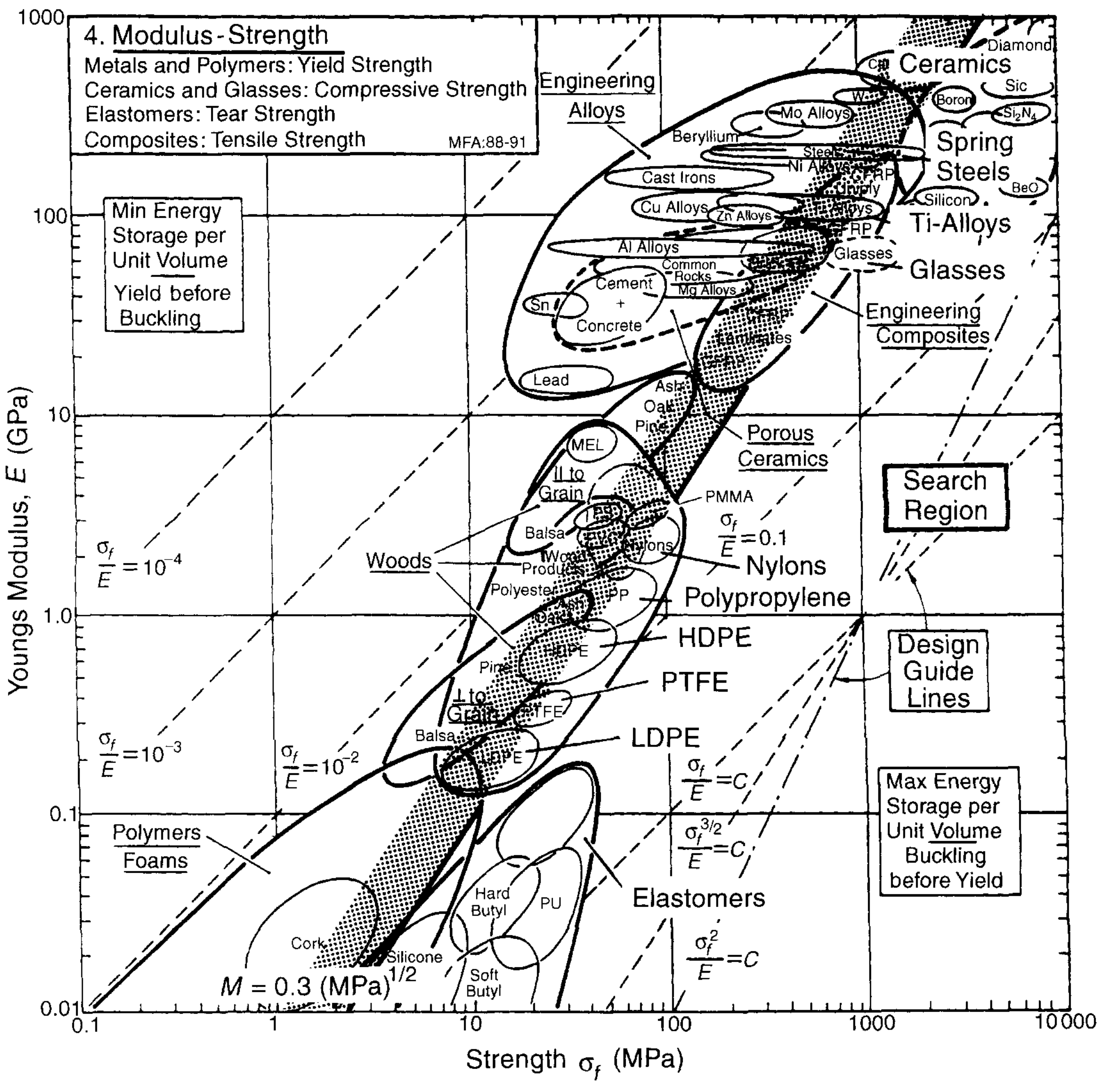

Fig. 6.24 Materials for elastic diaphragms. Elastomers, polymers, metals and even ceramics can be used; the final selection depends on details of the design.

Table 6.24 Materials for diaphragms

Material	$M = \frac{\sigma_f^{3/2}}{E}$ $(MPa)^{1/2}$	*Loss coefficient* η	*Comment*
Ceramics	0.3–3	$<10^{-4}$	Weak in tension. Eliminate.
Glasses	0.5	$\approx 10^{-4}$	Possible if protected from damage.
Spring Steel	0.3	$\approx 10^{-4}$	The standard choice. Low loss coefficient gives rapid response.
Ti-Alloys	0.3	$\approx 3 \times 10^{-4}$	As good as steel, corrosion resistant, expensive.
Nylons	0.3	$\approx 2 \times 10^{-2}$	Polymers creep and
Polypropylene	0.3	$\approx 5 \times 10^{-2}$	have high loss coefficients,
HDPE	0.3	$\approx 10^{-1}$	giving an actuator with
PTFE	0.3	$\approx 10^{-1}$	poor reproducibility.
Elastomers	0.5–10	$\approx 10^{-1}$–1	Excellent M value, giving large elastic deflection, but high loss coefficient limits response time.

Postscript

As always, application of the primary design criterion (large δ without failure) leads to a subset of materials to which further criteria are now applied. Elastomers have the best values of M, but they have high loss coefficients, are easily punctured, and may be permeable to certain gases or liquids. If corrosive liquids (sea water, cleaning fluids) may contact the diaphragm, then stainless steel or bronze may be preferable to a high-carbon steel, even though they have smaller values of M. This can be overcome by design: crimping the diaphragm or shaping it like a bellows magnifies deflection without increase in stress, but adding manufacturing cost.

Related case studies

Case Study 6.9: Materials for springs
Case Study 6.10: Elastic hinges
Case Study 6.11: Materials for seals
Case Study 6.13: Knife edges and pivots
Case Study 6.16: High damping materials for shaker tables

6.13 Knife edges and pivots

Middle-aged readers may remember the words '17 Sapphires' printed on the face of a watch, roughly where the word 'Quartz' now appears. A really expensive watch had, not sapphires, but diamonds. They are examples of good materials for knife edges and pivots. These are bearings in which two members are loaded together in nominal line or point contact, and can tilt relative to one another, or rotate freely about the load axis (Figure 6.25). The essential material properties, arising directly from the design requirements of Table 6.25, are high hardness (to carry the contact pressures) and high modulus (to give positional precision and to minimize frictional losses). But in what combination? And which materials have them?

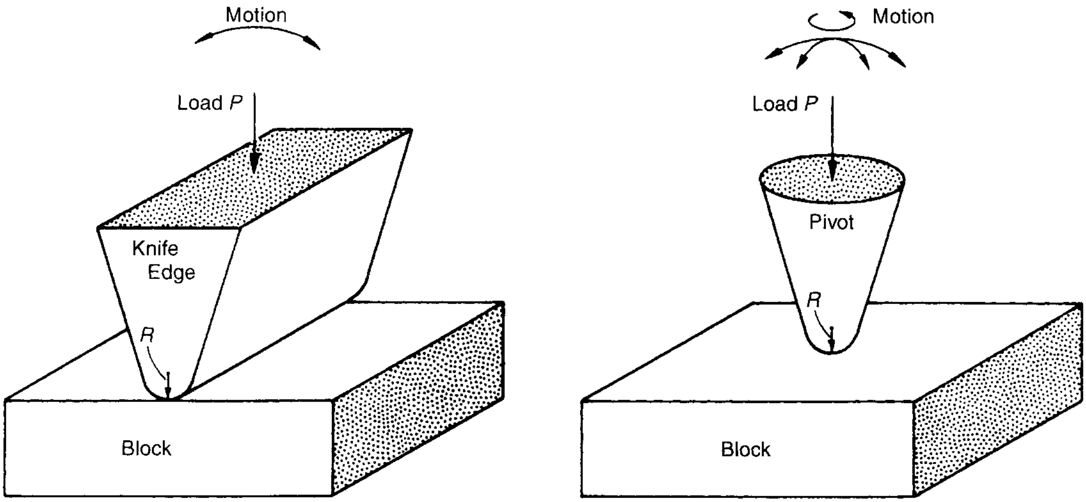

Fig. 6.25 A knife edge and a pivot. Good performance requires a high strength (to prevent plastic indentation or fracture) and a high modulus (to minimize elastic flattening at the contact which leads to frictional losses).

Table 6.25 Design requirements for knife edges and pivots

Function	Knife edges and pivots
Objective	(a) Maximize positional precision for given load, or (b) Maximize load capacity for given geometry
Constraints	(a) Contact stress must not damage either surface (b) Low thermal expansion (precision pivots) (c) High toughness (pivots exposed to shock loading)

The model

The first design goal is to maximize the load P that the contact can support, subject to the constraint that both faces of the bearing remain elastic. The contact pressure p at an elastic, non-conforming, contact (one which appears to touch at a point or along a line) is proportional to $(PE^2/R^2)^{1/3}$, where P is the load and R the radius of the knife-edge or pivot (Appendix A: 'Useful Solutions'). Check the dimensions: they are those of stress, MPa. Young's modulus, E, appears on the top because the elastic contact area decreases if E is large, and this increases the contact pressure. The knife or pivot will indent the block, or deform itself, if the contact pressure exceeds the hardness, H; and H is proportional to the strength, σ_f. The constraint is described by:

$$\left[\frac{PE^2}{R^2}\right]^{1/3} \leq C\sigma_f \tag{6.38}$$

where C is a constant (approximately 3.2). Thus, for a given geometry, the maximum bearing load is

$$P = C^3 R^2 \left[\frac{\sigma_f^3}{E^2}\right] \tag{6.39}$$

The subset of materials which maximizes the permitted bearing load is that with the greatest values of

$$M_1 = \frac{\sigma_f^3}{E^2}$$

The second constraint is that of low total contact area. The contact area A of any non-conforming contact has the form (Appendix A again)

$$A = C\left[\frac{PR}{E}\right]^{2/3} \tag{6.40}$$

where C is another constant (roughly 1). For any value of P less than that given by equation (6.39), this constraint is met by selecting from the subset those with the highest values of

$$M_2 = E$$

The selection

Once again, the material indices involve σ_f and E only. Chart 4 is shown in Figure 6.26. The two requirements isolate the top corner of the diagram and this time the loading is compressive, so ceramics are usable. Glasses, high-carbon steels and ceramics are all good choices. Table 6.26 gives more details: note the superiority of diamond.

Postscript

The final choice depends on the details of its application. In sensitive force balances and other measuring equipment, very low friction is important: then we need the exceptionally high modulus of sapphire or diamond. In high load-capacity devices (weigh bridges, mechanical testing equipment),

Table 6.26 Materials selection for knife edges and pivots

Material	$M = \frac{\sigma_f^3}{E^2}$ *(MPa)*	$M_2 = E$ *(GPa)*	*Comment*
Quartz	0.5	70	Good M_1 but brittle — poor impact resistance.
High-Carbon Steel	0.2	210	Some ductility, giving impact
Tool Steel	0.3	210	resistance; poor corrosion resistance.
Silicon	1	120	Good M_1, but brittle. Readily available in large quantities.
Sapphire, Al_2O_3	0.9	380	
Silicon Carbide, SiC	1	410	Excellent M_1 and M_2 with good corrosion
Silicon Nitride, Si_3N_4	1.1	310	resistance, but damaged by impact because of low toughness.
Tungsten Carbide, WC	1	580	
Diamond	2	1000	Outstanding on all counts except cost.

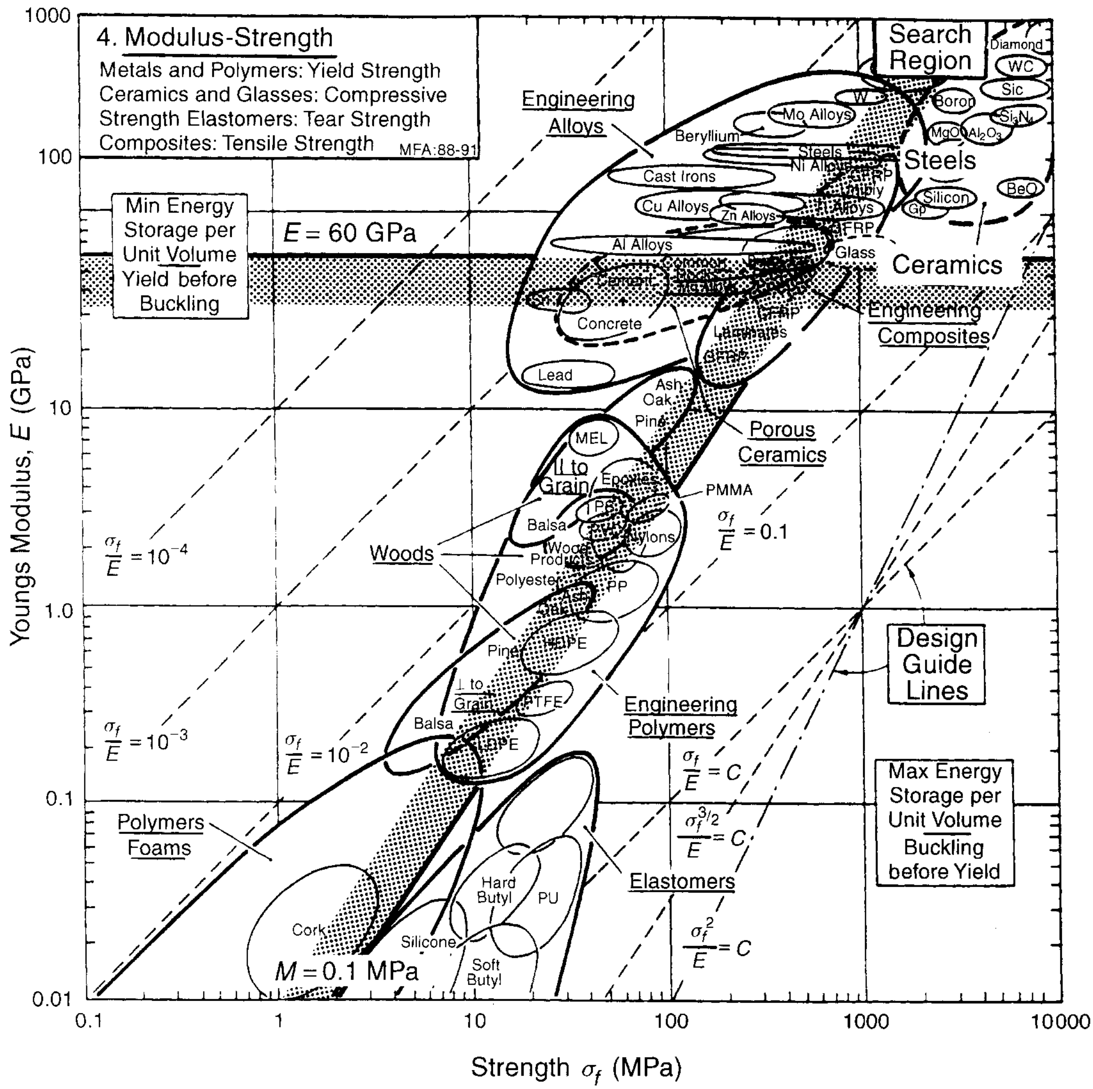

Fig. 6.26 Materials for knife edges and pivots. Ceramics, particularly diamond and silicon carbide, are good; fully hardened steel is a good choice too.

some ability to absorb overloads by limited plasticity is an advantage, and hardened steel is a good choice. If the environment is a potentially corrosive one — and this includes ordinary damp air — glass or a ceramic may be best. Note how the primary design criteria — high σ_f^3/E^2 and E — identify a subset from which, by considering further requirements, a single choice can be made.

Related case studies

Case Study 6.9: Materials for springs
Case Study 6.10: Elastic hinges

Case Study 6.11: Materials for seals
Case Study 6.12: Diaphragms for pressure actuators
Case Study 6.20: Minimizing distortion in precision devices
Case Study 6.21: Ceramic valves for taps

6.14 Deflection-limited design with brittle polymers

Among mechanical engineers there is a rule-of-thumb: avoid materials with fracture toughnesses K_{Ic} less than $15\,\text{MPa}\,\text{m}^{1/2}$. Almost all metals pass: they have values of K_{Ic} in the range of 20–100 in these units. White cast iron, and a few powder metallurgy products fail; they have values around $10\,\text{MPa}\,\text{m}^{1/2}$. Ordinary engineering ceramics have fracture toughnesses in the range $1-6\,\text{MPa}\,\text{m}^{1/2}$; mechanical engineers view them with deep suspicion. But engineering polymers are even less tough, with K_{Ic} values in the range $0.5-3\,\text{MPa}\,\text{m}^{1/2}$, and yet engineers use them all the time. What is going on here?

When a brittle material is deformed, it deflects elastically until it fractures. The stress at which this happens is

$$\sigma_f = \frac{CK_c}{\sqrt{\pi a_c}} \tag{6.41}$$

where K_c is an appropriate fracture toughness, a_c is the length of the largest crack contained in the material and C is a constant which depends on geometry, but is usually about 1. In a *load-limited* design — a tension member of a bridge, say — the part will fail in a brittle way if the stress exceeds that given by equation (6.41). Here, obviously, we want materials with high values of K_c.

But not all designs are load limited; some are *energy limited*, others are *deflection limited*. Then the criterion for selection changes. Consider, then, the three scenarios created by the three alternative constraints of Table 6.27.

The model

In *load-limited* design the component must carry a specified load or pressure without fracturing. Then the local stress must not exceed that specified by equation (6.41) and, for minimum volume, the best choice of materials are those with high values of

$$\boxed{M_1 = K_c} \tag{6.42}$$

Table 6.27 Design requirements for

Function	Resist brittle fracture
Objective	Minimize volume (mass, cost...)
Constraints	(a) Design load specified or
	(b) Design energy specified or
	(c) Design deflection specified

It is usual to identify K_c with the plane-strain fracture toughness, corresponding to the most highly constrained cracking conditions, because this is conservative. For load-limited design using thin sheet, a plane-stress fracture toughness may be more appropriate; and for multi-layer materials, it may be an interface fracture toughness that matters. The point, though, is clear enough: the best materials for load-limited design are those with large values of appropriate K_c.

But, as we have said, not all design is load limited. Springs, and containment systems for turbines and flywheels are *energy* limited. Take the spring (Figure 6.16) as an example. The elastic energy per unit volume stored in the spring is the integral over the volume of

$$U_e = \frac{1}{2}\sigma\varepsilon = \frac{1}{2}\frac{\sigma^2}{E}$$

The stress is limited by the fracture stress of equation (6.41) so that — if 'failure' means 'fracture' — the maximum energy the spring can store is

$$U_e^{\max} = \frac{C^2}{2\pi a_c}\left(\frac{K_{Ic}^2}{E}\right)$$

For a given initial flaw size, energy is maximized by choosing materials with large values of

$$\boxed{M_2 = \frac{K_{Ic}^2}{E} \approx J_c} \qquad (6.43)$$

where J_c is the toughness (usual units: kJ/m^2).

There is a third scenario: that of *displacement*-limited design (Figure 6.27). Snap-on bottle tops, snap together fasteners and such like are displacement limited: they must allow sufficient elastic displacement to permit the snap-action without failure, requiring a large failure strain ε_f. The strain is related to the stress by Hooke's law

$$\varepsilon = \frac{\sigma}{E}$$

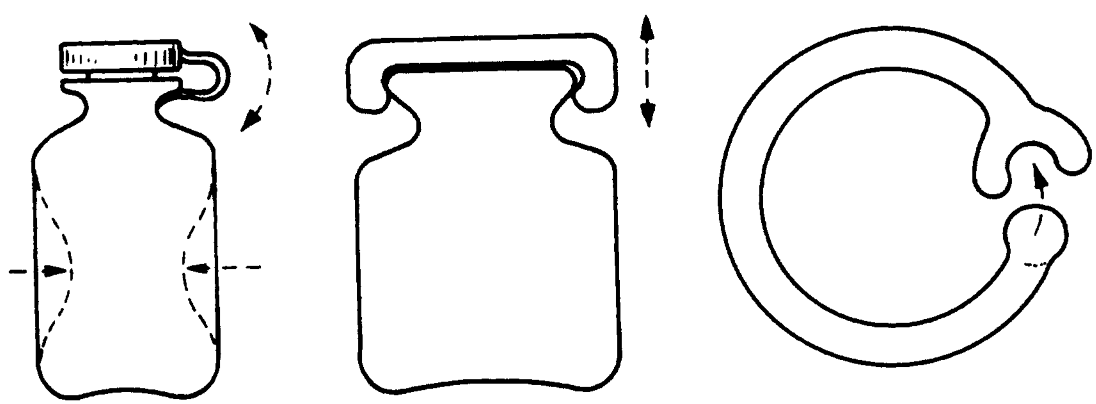

Fig. 6.27 Load and deflection-limited design. Polymers, having low moduli, frequently require deflection-limited design methods.

and the stress is limited by the fracture equation (6.41). Thus the failure strain is

$$\varepsilon_f = \frac{CK_{Ic}}{\sqrt{\pi a_c}E}$$

The best materials for displacement-limited design are those with large values of

$$\boxed{M_3 = \frac{K_{Ic}}{E}}$$

The selection

Figure 6.28 shows a chart of fracture toughness, K_{Ic}, plotted against modulus E. It allows materials to be compared by values of fracture toughness, M_1 , by toughness, M_2, and by values of the deflection-limited index M_3. As the engineer's rule-of-thumb demands, almost all metals have values of K_{Ic} which lie above the 15 MPa $m^{1/2}$ acceptance level for load-limited design. Polymers and ceramics do not.

The line showing M_2 on Figure 6.28 is placed at the value 1 kJ/m^2. Materials with values of M_2 greater than this have a degree of shock-resistance with which engineers feel comfortable (another rule-of-thumb). Metals, composites and some polymers qualify (Table 6.28); ceramics do not. When we come to deflection-limited design, the picture changes again. The line shows the index $M_3 = K_{Ic}/E$ at the value 10^{-3} $m^{1/2}$. It illustrates why polymers find such wide application: when the design is deflection limited, polymers — particularly nylons, polycarbonates and polystyrene — are as good as the best metals.

Postscript

The figure gives further insights. The mechanical engineers' love of metals (and, more recently, of composites) is inspired not merely by the appeal of their K_{Ic} values. They are good by all three criteria (K_{Ic}, K_{Ic}^2/E and K_{Ic}/E). Polymers have good values of K_{Ic}/E but not the other two. Ceramics are poor by all three criteria. Herein lie the deeper roots of the engineers' distrust of ceramics.

Further reading

Background in fracture mechanics and safety criteria can be found in these books:

Brock, D. (1984) *Elementary Engineering Fracture Mechanics*, Martinus Nijoff, Boston.
Hellan, K. (1985) *Introduction to Fracture Mechanics*, McGraw-Hill.
Hertzberg, R.W. (1989) *Deformation and Fracture Mechanics of Engineering Materials*, Wiley, New York.

Related case studies

Case Study 6.9: Materials for springs
Case Study 6.10: Elastic hinges and couplings
Case Study 6.15: Safe pressure vessels

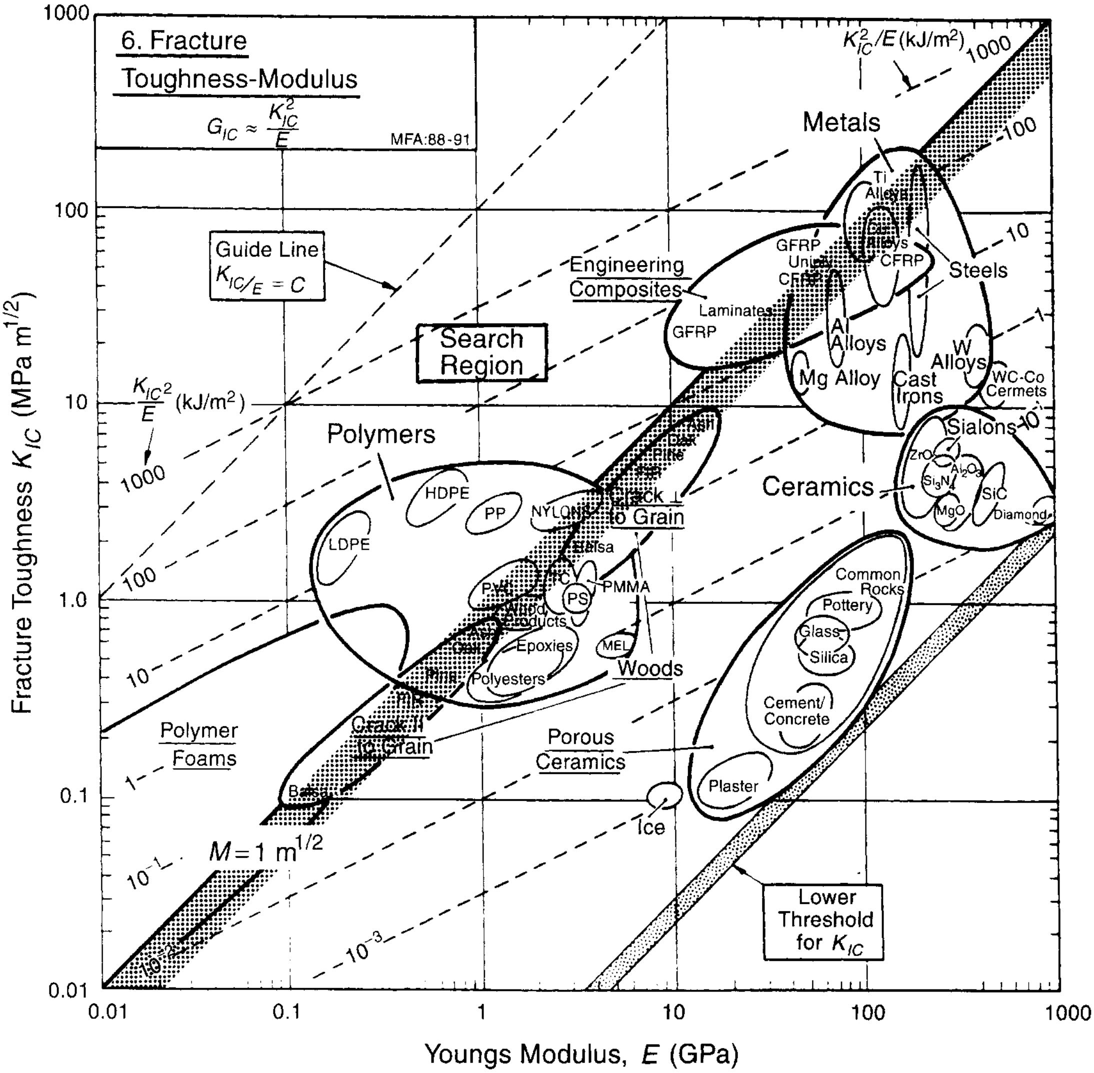

Fig. 6.28 The selection of materials for load, deflection and energy-limited design. In deflection-limited design, polymers are as good as metals, despite having very low values of fracture toughness.

Table 6.28 Materials for fracture-limited design

Design type, and rule-of-thumb	*Material*
Load-limited design $K_{Ic} > 15\,\mathrm{MPa\,m^{1/2}}$	Metals, polymer-matrix composites.
Energy-limited design $J_c > 1\,\mathrm{kJ/m^2}$	Metals, composites and some polymers.
Displacement-limited design $K_{Ic}/E > 10^{-3}\,\mathrm{m^{1/2}}$	Polymers, elastomers and some metals.

6.15 Safe pressure vessels

Pressure vessels, from the simplest aerosol-can to the biggest boiler, are designed, for safety, to yield or leak before they break. The details of this design method vary. Small pressure vessels are usually designed to allow general yield at a pressure still too low to cause any crack the vessel may contain to propagate ('yield before break'); the distortion caused by yielding is easy to detect and the pressure can be released safely. With large pressure vessels this may not be possible. Instead, safe design is achieved by ensuring that the smallest crack that will propagate unstably has a length greater than the thickness of the vessel wall ('leak before break'); the leak is easily detected, and it releases pressure gradually and thus safely (Table 6.29). The two criteria lead to different material indices. What are they?

The model

The stress in the wall of a thin-walled spherical pressure vessel of radius R (Figure 6.29) is

$$\sigma = \frac{pR}{2t} \tag{6.45}$$

In pressure vessel design, the wall thickness, t, is chosen so that, at the working pressure p, this stress is less than the yield strength, σ_f, of the wall. A small pressure vessel can be examined

Table 6.29 Design requirements for safe pressure vessels

Function	Pressure vessel = contain pressure, p
Objective	Maximum safety
Constraints	(a) Must yield before break or (b) Must leak before break (c) Wall thickness small to reduce mass and cost

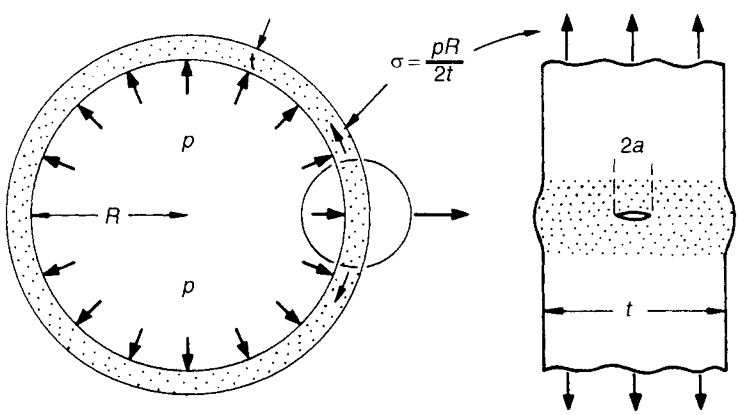

Fig. 6.29 A pressure vessel containing a flaw. Safe design of small pressure vessels requires that they yield before they break; that of large pressure vessels may require, instead, that they leak before they break.

ultrasonically, or by X-ray methods, or proof tested, to establish that it contains no crack or flaw of diameter greater than $2a_c$; then the stress required to make the crack propagate* is

$$\sigma = \frac{CK_{Ic}}{\sqrt{\pi a_c}} \tag{6.46}$$

where C is a constant near unity. Safety can be achieved by ensuring that the working stress is less than this; but greater security is obtained by requiring that the crack will not propagate even if the stress reaches the general yield stress — for then the vessel will deform stably in a way which can be detected. This condition is expressed by setting σ equal to the yield stress, σ_f, giving

$$\pi a_c \leq C^2 \left[\frac{K_{Ic}}{\sigma_f}\right]^2$$

The tolerable crack size is maximized by choosing a material with the largest value of

$$\boxed{M = \frac{K_{Ic}}{\sigma_f}}$$

Large pressure vessels cannot always be X-rayed or sonically tested; and proof testing them may be impractical. Further, cracks can grow slowly because of corrosion or cyclic loading, so that a single examination at the beginning of service life is not sufficient. Then safety can be ensured by arranging that a crack just large enough to penetrate both the inner and the outer surface of the vessel is still stable, because the leak caused by the crack can be detected. This is achieved if the stress is always less than or equal to

$$\sigma = \frac{CK_{Ic}}{\sqrt{\pi t/2}} \tag{6.47}$$

The wall thickness t of the pressure vessel was, of course, designed to contain the pressure p without yielding. From equation (6.45), this means that

$$t \geq \frac{pR}{2\sigma_f} \tag{6.48}$$

Substituting this into the previous equation (with $\sigma = \sigma_f$) gives

$$C^2 \frac{\pi pR}{4} = \left[\frac{K_{Ic}^2}{\sigma_f}\right] \tag{6.49}$$

The maximum pressure is carried most safely by the material with the greatest value of

$$\boxed{M_2 = \frac{K_{Ic}^2}{\sigma_f}}$$

* If the wall is sufficiently thin, and close to general yield, it will fail in a plane-stress mode. Then the relevant fracture toughness is that for plane stress, not the smaller value for plane strain.

Both M_1 and M_2 could be made large by making the yield strength of the wall, σ_f, very small: lead, for instance, has high values of both, but you would not choose it for a pressure vessel. That is because the vessel wall must also be as thin as possible, both for economy of material, and to keep it light. The thinnest wall, from equation (6.48), is that with the largest yield strength, σ_f. Thus we wish also to maximize

$$\boxed{M_3 = \sigma_f}$$

narrowing further the choice of material.

The selection

These selection criteria are applied by using the chart shown in Figure 6.30: the fracture toughness, K_{Ic}, plotted against strength σ_f. The three criteria appear as lines of slope 1, 1/2 and as lines that are vertical. Take 'yield before break' as an example. A diagonal line corresponding to $M = K_{Ic}/\sigma_f = C$ links materials with equal performance; those above the line are better. The line shown in the figure at $M_1 = 0.6\,\text{m}^{1/2}$ excludes everything but the toughest steels, copper and aluminium alloys, though some polymers nearly make it (pressurized lemonade and beer containers are made of these polymers). A second selection line at $M_3 = 100\,\text{MPa}$ eliminates aluminium alloys. Details are given in Table 6.30.

Large pressure vessels are always made of steel. Those for models (a model steam engine, for instance) are copper; it is favoured in the small-scale application because of its greater resistance to corrosion. The reader may wish to confirm that the alternative criterion

$$\boxed{M_2 = \frac{K_{Ic}^2}{\sigma_f}}$$

favours steel more strongly, but does not greatly change the conclusions.

Postscript

Boiler failures used to be common place — there are even songs about it. Now they are rare, though when safety margins are pared to a minimum (rockets, new aircraft designs) pressure vessels still

Table 6.30 Materials for safe pressure vessels

Material	$M_1 = \frac{K_{Ic}}{\sigma_f}$ $(m^{1/2})$	$M_3 = \sigma_f$ *(MPa)*	*Comment*
Tough steels	>0.6	300	These are the pressure-vessel steels, standard in this application.
Tough copper alloys	>0.6	120	OFHC Hard drawn copper.
Tough Al-alloys	>0.6	80	1000 and 3000 series Al-alloys.
Ti-alloys	02	700	High yield but low
High-strength Al-alloys	0.1	500	safety margin. Good for light
GFRP/CFRP	0.1	500	pressure vessels.

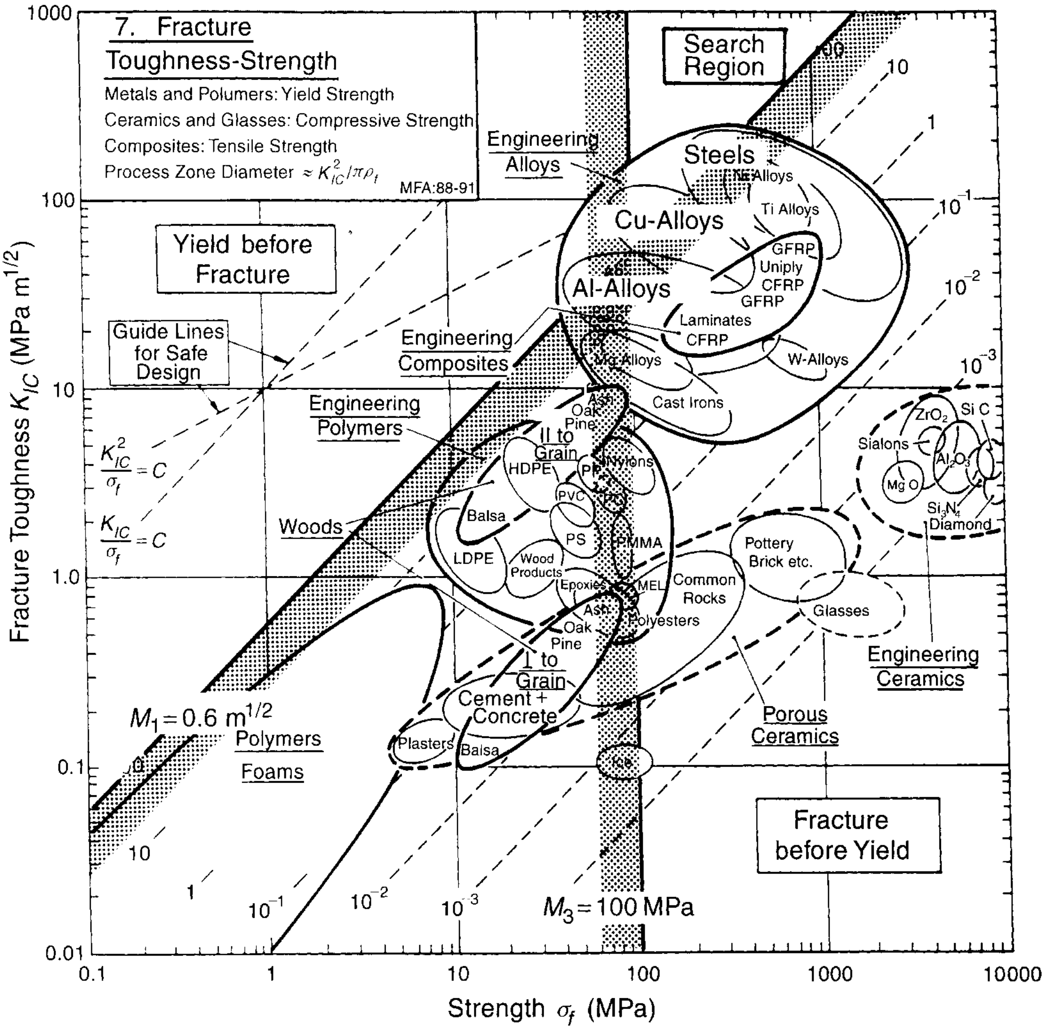

Fig. 6.30 Materials for pressure vessels. Steel, copper alloys and aluminium alloys best satisfy the 'yield before break' criterion. In addition, a high yield strength allows a high working pressure. The materials in the 'search area' triangle are the best choice. The leak-before-break criterion leads to essentially the same selection.

occasionally fail. This (relative) success is one of the major contributions of fracture mechanics to engineering practice.

Further reading

Background in fracture mechanics and safety criteria can be found in these books:

Brock, D. (1984) *Elementary Engineering Fracture Mechanics*, Martinus Nijoff, Boston.
Hellan, K. (1985) *Introduction to Fracture Mechanics*, McGraw-Hill.
Hertzberg, R.W. (1989) *Deformation and Fracture Mechanics of Engineering Materials*, Wiley, New York.

Related case studies

Case Study 6.6: Materials for flywheels
Case Study 6.14: Deflection-limited design with brittle polymers

6.16 Stiff, high damping materials for shaker tables

Shakers, if you live in Pennsylvania, are the members of an obscure and declining religious sect, noted for their austere wooden furniture. To those who live elsewhere they are devices for vibration testing. This second sort of shaker consists of an electromagnetic actuator driving a table, at frequencies up to 1000 Hz, to which the test-object (a space probe, an automobile, an aircraft component or the like) is clamped (Figure 6.31). The shaker applies a spectrum of vibration frequencies, f, and amplitudes, A, to the test-object to explore its response.

A big table operating at high frequency dissipates a great deal of power. The primary objective is to minimize this, but subject to a number of constraints itemized in Table 6.31. What materials make good shaker tables?

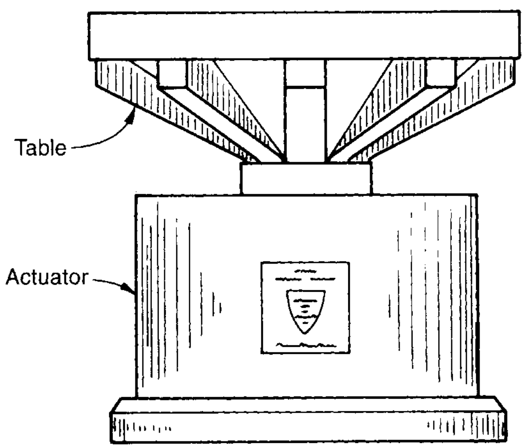

Fig. 6.31 A shaker table. It is required to be stiff, but have high intrinsic 'damping' or loss coefficient.

Table 6.31 Design requirements for shaker tables

Function	Table for vibration tester (shaker table)
Objective	Minimize power consumption
Constraints	(a) Radius, R, specified
	(b) Must be stiff enough to avoid distortion by clamping forces
	(c) Natural frequencies above maximum operating frequency (to avoid resonance)
	(d) High damping to minimize stray vibrations
	(e) Tough enough to withstand mishandling and shock

The model

The power p (watts) consumed by a dissipative vibrating system with a sinusoidal input is equal to

$$p = C_1 m A^2 \omega^3$$

where m is the mass of the table, A is the amplitude of vibration, ω is the frequency (rad/s) and C_1 is a constant. Provided the operating frequency ω is significantly less than the resonant frequency of the table, then $C_1 \approx 1$. The amplitude A and the frequency ω are prescribed. To minimize the power lost in shaking the table itself, we must minimize its mass m. We idealize the table as a disc of given radius, R. Its thickness, t, is a free variable which we may choose. Its mass is

$$m = \pi R^2 t \rho \tag{6.49}$$

where ρ is the density of the material of which it is made. The thickness influences the bending-stiffness of the table — and this is important both to prevent the table flexing too much under clamping loads, and because it determines its lowest natural vibration frequency. The bending stiffness, S, is

$$S = \frac{C_2 E I}{R^3},$$

where C_2 is a constant. The second moment of the section, I, is proportional to $t^3 R$. Thus, for a given stiffness S and radius R,

$$t = C_3 \left(\frac{S R^2}{E} \right)^{1/3}$$

where C_3 is another constant. Inserting this into equation (6.49) we obtain

$$m = C_3 \pi R^{8/3} S^{1/3} \frac{\rho}{E^{1/3}}$$

The mass of the table, for a given stiffness and minimum vibration frequency, is therefore minimized by selecting materials with high values of

$$\boxed{M_1 = \frac{E^{1/3}}{\rho}}$$

There are three further requirements. The first is that of high mechanical damping η. The second that the fracture toughness K_{Ic} of the table be sufficient to withstand mishandling and clamping forces. And the third is that the material should not cost too much.

The selection

Figure 6.32 shows Chart 8: loss coefficient η plotted against modulus E. The vertical line shows the constraint $E \geq 30\,\text{GPa}$, the horizontal one, the constraint $\eta > 0.01$. The search region contains several suitable materials, notably magnesium, cast iron, various composites and concrete (Table 6.32). Of these, magnesium and composites have high values of $E^{1/3}/\rho$, and both have low densities. Among metals, magnesium is the best choice; otherwise GFRP.

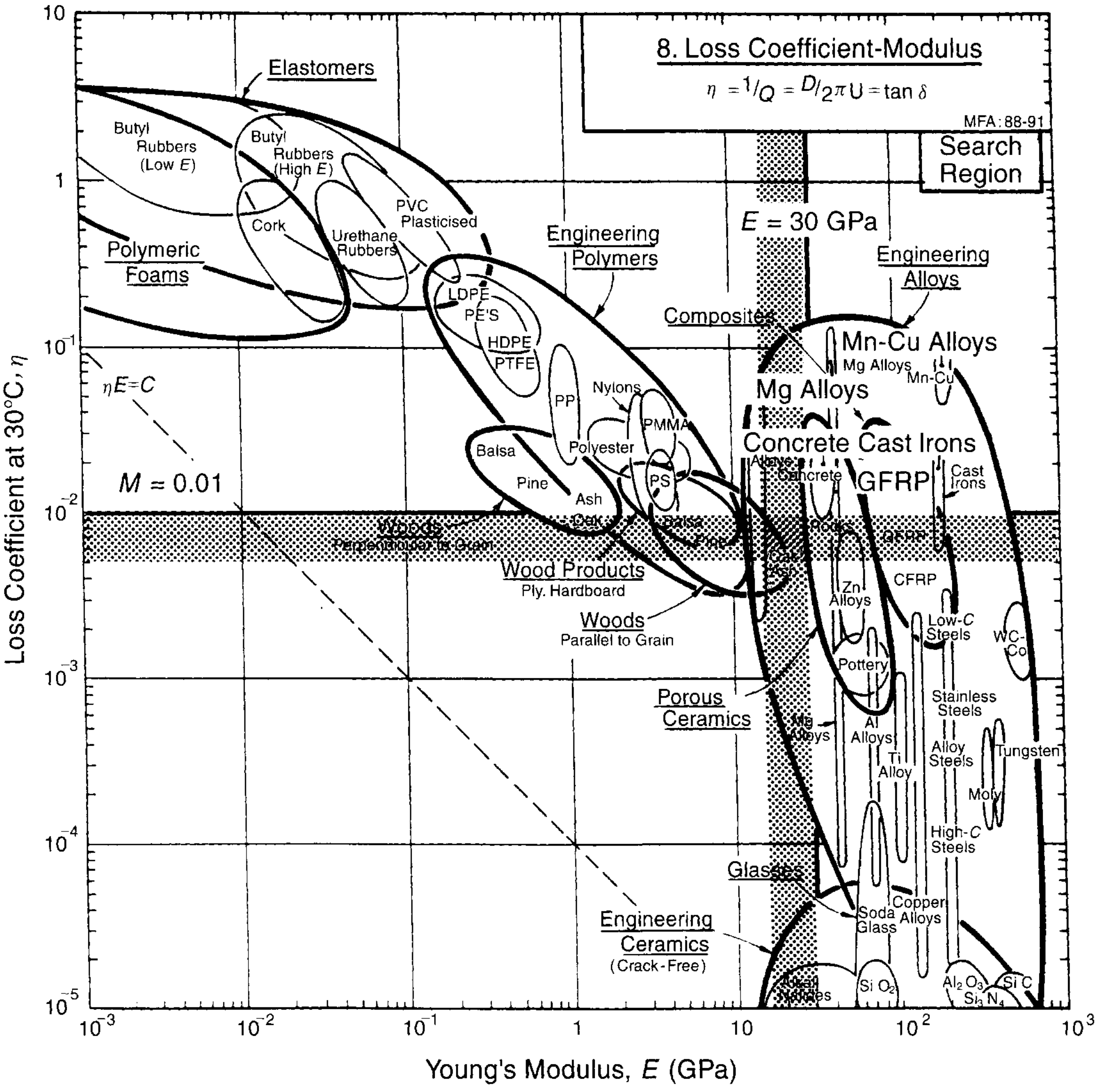

Fig. 6.32 Selection of materials for the shaker table. Magnesium alloys, cast irons, GFRP, concrete and the special high-damping Mn–Cu alloys are candidates.

Postscript

Stiffness, high natural frequencies and damping are qualities often sought in engineering design. The shaker table found its solution (in real life as well as this case study) in the choice of a cast magnesium alloy.

Sometimes, a solution is possible by combining materials. The loss coefficient chart shows that polymers and elastomers have high damping. Sheet steel panels, prone to lightly-damped vibration, can be damped by coating one surface with a polymer, a technique exploited in automobiles, typewriters and machine tools. Aluminium structures can be stiffened (raising natural frequencies) by bonding carbon fibre to them: an approach sometimes used in aircraft design. And structures

Table 6.32 Materials for shaker tables

Material	*Loss coeff., η*	$M = E^{1/3}/\rho$	ρ *(Mg/m)*^{3}	*Comment*
Mg-alloys	$10^{-2}-10^{-1}$		1.75	The best combination of properties.
Mn–Cu alloys	10^{-1}		8.0	Good damping but heavy.
KFRP/GFRP	2×10^{-2}		1.8	Less damping than Mg-alloys, but possible.
Cast irons	2×10^{-2}		7.8	Good damping but heavy.
Concrete	2×10^{-2}		2.5	Less damping than Mg-alloys, but possible for a large table.

loaded in bending or torsion can be made lighter, for the same stiffness (again increasing natural frequencies), by shaping them efficiently: by attaching ribs to their underside, for instance. Shaker tables — even the austere wooden tables of the Pennsylvania Shakers — exploit shape in this way.

Further reading

Tustin, W. and Mercado, R. (1984) *Random Vibrations in Perspective.* Tustin Institute of Technology Inc, Santa Barbara, CA, USA.

Cebon, D. and Ashby, M.F. (1994) Materials selection for precision instruments, *Meas. Sci. and Technol.*, Vol. 5, pp. 296–306.

Related case studies

Case Study 6.4: Materials for table legs
Case Study 6.9: Materials for springs
Case Study 6.12: Diaphragms for pressure actuators
Case Study 6.20: Minimizing distortion in precision devices

6.17 Insulation for short-term isothermal containers

Each member of the crew of a military aircraft carries, for emergencies, a radio beacon. If forced to eject, the crew member could find himself in trying circumstances — in water at 4°C, for example (much of the earth's surface is ocean with a mean temperature of roughly this). The beacon guides friendly rescue services, minimizing exposure time.

But microelectronic metabolisms (like those of humans) are upset by low temperatures. In the case of the beacon, it is its transmission frequency which starts to drift. The design specification for the egg-shaped package containing the electronics (Figure 6.33) requires that, when the temperature of the outer surface is changed by 30°C, the temperature of the inner surface should not change significantly for an hour. To keep the device small, the wall thickness is limited to a thickness w of 20 mm. What is the best material for the package? A dewar system is out — it is too fragile.

A foam of some sort, you might think. But here is a case in which intuition leads you astray. So let us formulate the design requirements (Table 6.33) and do the job properly.

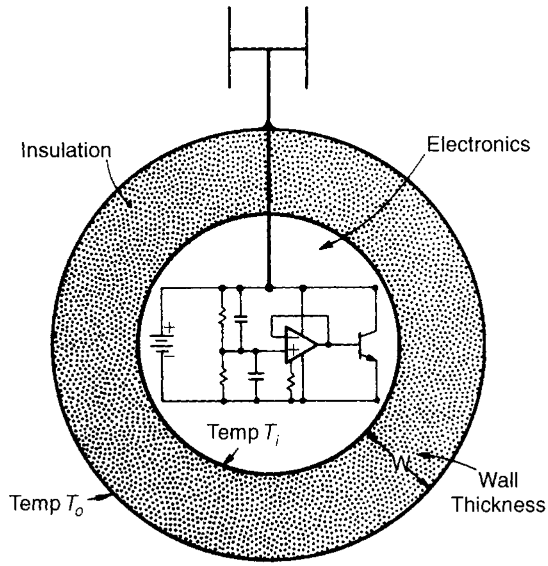

Fig. 6.33 An isothermal container. It is designed to maximize the time before the inside temperature changes after the outside temperature has suddenly changed.

Table 6.33 Design requirements for short-term insulation

Function	Short-term thermal insulation
Objective	Maximize time t before internal temperature of container falls appreciably when external temperature suddenly drops
Constraints	Wall thickness must not exceed w

The model

We model the container as a wall of thickness w, thermal conductivity λ. The heat flux J through the wall, once a steady-state has been established, is

$$J = \lambda \frac{(T_i - T_o)}{w} \tag{6.50}$$

where T_o is the temperature of the outer surface and T_i that of the inner one (Figure 6.33). The only free variable here is the thermal conductivity, λ. The flux is minimized by choosing a wall material with the lowest possible value of λ. Chart 9 (Figure 6.34) shows that this is, indeed, a foam.

But we have answered the wrong question. The design brief was not to minimize the heat flux, but the *time* before the temperature of the inner wall changed appreciably. When the surface temperature of a body is suddenly changed, a temperature wave, so to speak, propagates inwards. The distance x it penetrates in time t is approximately $\sqrt{2at}$. Here a is the thermal diffusivity, defined by $a = \lambda/\rho C_p$, where ρ is the density and C_p is the specific heat (Appendix A: 'Useful Solutions'). Equating this to the wall thickness w gives

$$t \approx \frac{w^2}{2a} \tag{6.51}$$

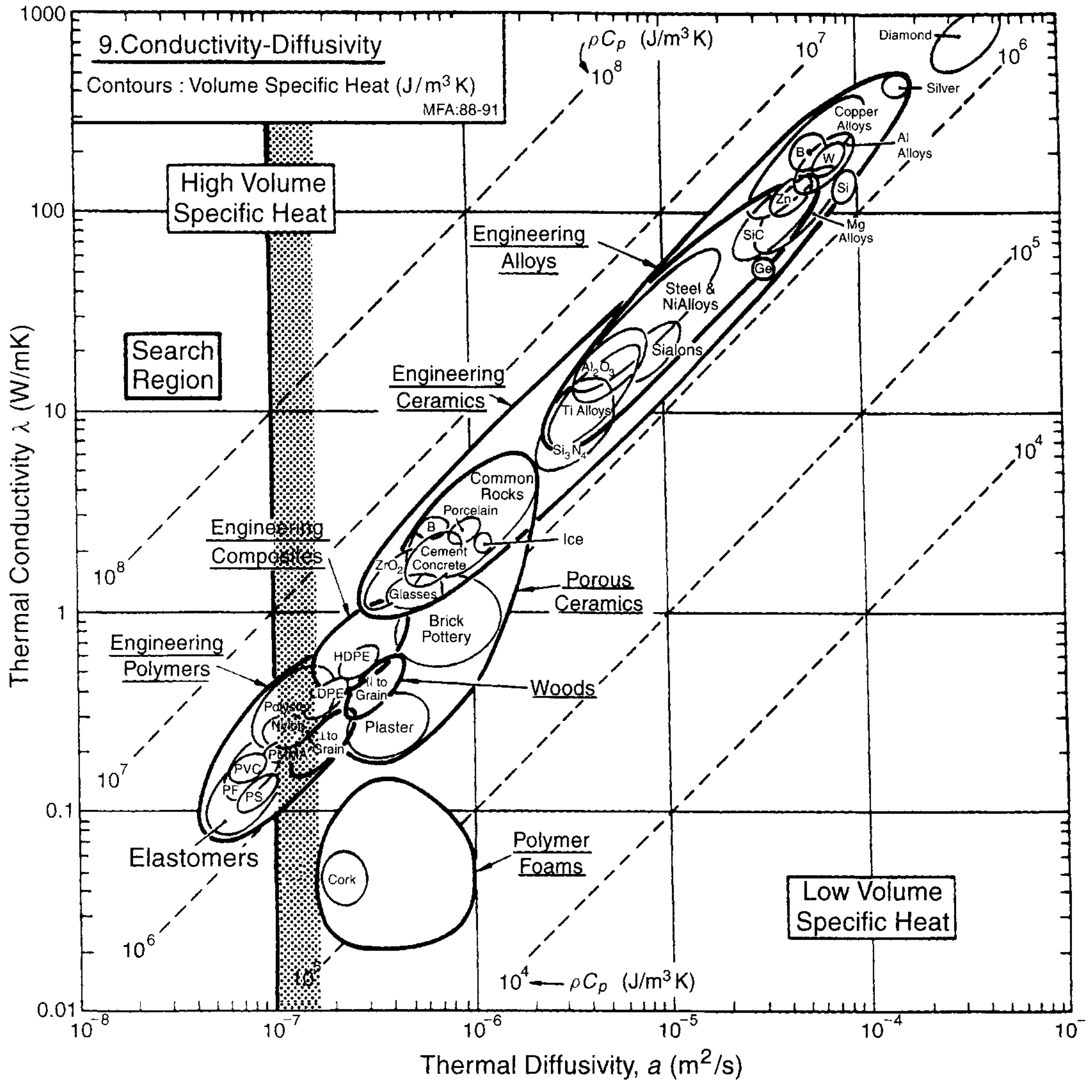

Fig. 6.34 Materials for short-term isothermal containers. Elastomers are good; foams are not.

The time is maximized by choosing the smallest value of the thermal diffusivity, a, not the conductivity λ.

The selection

Chart 9 (Figure 6.34) shows that the thermal diffusivities of foams are not particularly low; it is because they have so little mass, and thus heat capacity. The diffusivity of heat in a solid polymer or elastomer is much lower because they have specific heats which are particularly large. A package made of solid rubber, polystyrene or nylon, would — if of the same thickness — give the beacon a transmission life 10 times greater than one made of (say) a polystyrene foam, although of course

Table 6.34 Materials for short-term thermal insulation

Material	*Comment*
Elastomers: Butyl rubber (BR), Polychloroprene (CR), and Chlorosulfinated polyethylene (CSM) are examples	Best choice for short-term insulation.
Commodity polymers: Polyethylenes and Polypropylenes	Cheaper than elastomers, but somewhat less good for short-term insulation.
Polymer foams	Much less good than elastomers for short-term insulation; best choice for long-term insulation at steady state.

it would be heavier. The reader can confirm that 22 mm of a solid elastomer ($a = 7 \times 10^{-8}\,\text{m}^2/\text{s}$, read from Chart 9) will allow a time interval of 1 hour after an external temperature change before the internal temperature shifts much. Table 6.34 summarizes the results of materials selection.

Postscript

One can do better than this. The trick is to exploit other ways of absorbing heat. If a liquid — a low-melting wax, for instance — can be found that solidifies at a temperature equal to the minimum desired operating temperature for the transmitter (T_i), it can be used as a 'latent-heat sink'. Channels in the package are filled with the liquid; the inner temperature can only fall below the desired operating temperature when all the liquid has solidified. The latent heat of solidification must be supplied to do this, giving the package a large (apparent) specific heat, and thus an exceptionally low diffusivity for heat at the temperature T_i. The same idea is, in reverse, used in 'freezer packs' which solidify when placed in the freezer compartment of a refrigerator and remain cold (by melting, at 4°C) when packed around warm beer cans in a portable cooler.

Further reading

Holman, J.P. (1981) *Heat Transfer*, 5th edition, McGraw-Hill, New York.

Related case studies

Case Study 6.18: Energy-efficient kiln walls
Case Study 6.19: Materials for heat-storing walls

6.18 Energy-efficient kiln walls

The energy cost of one firing cycle of a large pottery kiln (Figure 6.35) is considerable. Part is the cost of the energy which is lost by conduction through the kiln walls; it is reduced by choosing a wall material with a low conductivity, and by making the wall thick. The rest is the cost of the energy used to raise the kiln to its operating temperature; it is reduced by choosing a wall material with a low heat capacity, and by making the wall thin. Is there a material index which captures these apparently conflicting design goals? And if so, what is a good choice of material for kiln walls? The choice is based on the requirements of Table 6.35.

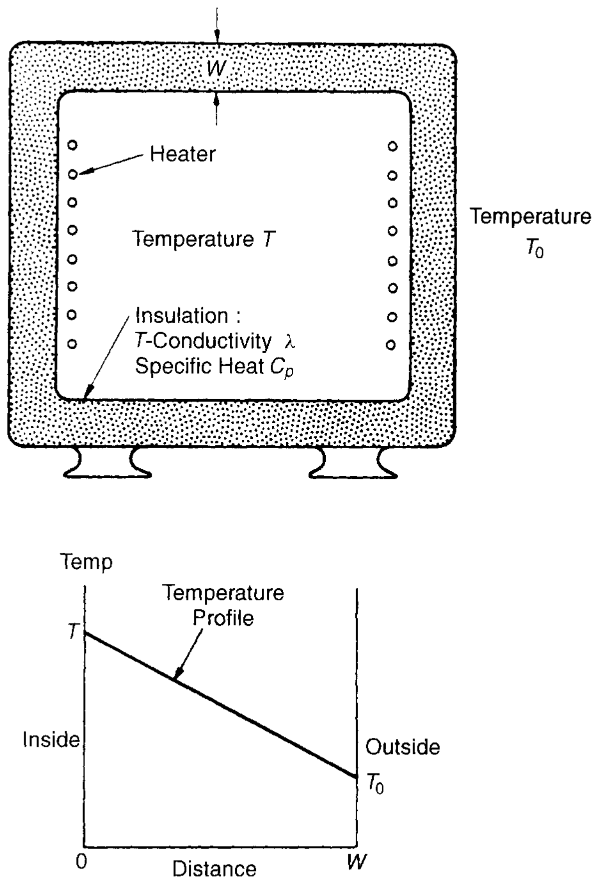

Fig. 6.35 A kiln. On firing, the kiln wall is first heated to the operating temperature, then held at this temperature. A linear gradient is then expected through the kiln wall.

Table 6.35 Design requirements for kiln walls

Function	Thermal insulation for kiln (cyclic heating and cooling)
Objective	Minimized energy consumed in firing cycle
Constraints	(a) Maximum operating temperature 1000 K (b) Possible limit on kiln-wall thickness for space reasons

The model

When a kiln is fired, the temperature rises quickly from ambient, T_o, to the operating temperature, T_i, where it is held for the firing time t. The energy consumed in the firing time has, as we have said, two contributions. The first is the heat conducted out: at steady state the heat loss by conduction, Q_1, per unit area, is given by the first law of heat flow. If held for time t it is

$$Q_1 = -\lambda \frac{dT}{dx} t = \lambda \frac{(T_i - T_o)}{w} t \qquad (6.52)$$

Here λ is the thermal conductivity, dT/dx is the temperature gradient and w is the insulation wall-thickness. The second contribution is the heat absorbed by the kiln wall in raising it to T_i, and this can be considerable. Per unit area, it is

$$Q_2 = C_p \rho w \left(\frac{T_i - T_o}{2} \right) \tag{6.53}$$

where C_p is the specific heat of the wall material and ρ is its density. The total energy consumed per unit area is the sum of these two:

$$Q = Q_1 + Q_2 = \frac{\lambda(T_i - T_o)t}{w} + \frac{C_p \rho w (T_i - T_o)}{2} \tag{6.54}$$

A wall which is too thin loses much energy by conduction, but absorbs little energy in heating the wall itself. One which is too thick does the opposite. There is an optimum thickness, which we find by differentiating equation (6.54) with respect to wall thickness w and equating the result to zero, giving:

$$w = \left(\frac{2\lambda t}{C_p \rho} \right)^{1/2} = (2at)^{1/2} \tag{6.55}$$

where $a = \lambda / C_p \rho$ is the thermal diffusivity. The quantity $(2at)^{1/2}$ has dimensions of length and is a measure of the distance heat can diffuse in time t. Equation (6.55) says that the most energy-efficient kiln wall is one that only starts to get really hot on the outside as the firing cycle approaches completion. Substituting equation (6.55) back into equation (6.54) to eliminate w gives:

$$Q = (T_i - T_o)(2t)^{1/2}(\lambda C_p \rho)^{1/2}$$

Q is minimized by choosing a material with a low value of the quantity $(\lambda C_p \rho)^{1/2}$, that is, by maximizing

$$\boxed{M_1 = (\lambda C_p \rho)^{-1/2} = \frac{a^{1/2}}{\lambda}} \tag{6.56}$$

But, by eliminating the wall thickness w we have lost track of it. It could, for some materials, be excessively large. We must limit it. A given firing time, t, and wall thickness, w, defines, via equation (6.55), an upper limit for the thermal diffusivity, a:

$$a \leq \frac{w^2}{2t}$$

Selecting materials which maximize equation (6.56) with the constraint on a defined by the last equation minimizes the energy consumed per firing cycle.

The selection

Figure 6.36 shows the λ–a chart with a selection line corresponding to $M = a^{1/2}/\lambda$ plotted on it. Polymer foams, cork and solid polymers are good, but only if the internal temperature is less than 100°C. Real kilns operate near 1000°C. Porous ceramics are the obvious choice (Table 6.36). Having chosen a material, the acceptable wall thickness is calculated from equation (6.55). It is listed, for a firing time of 3 hours (approximately 10^4 seconds) in Table 6.35.

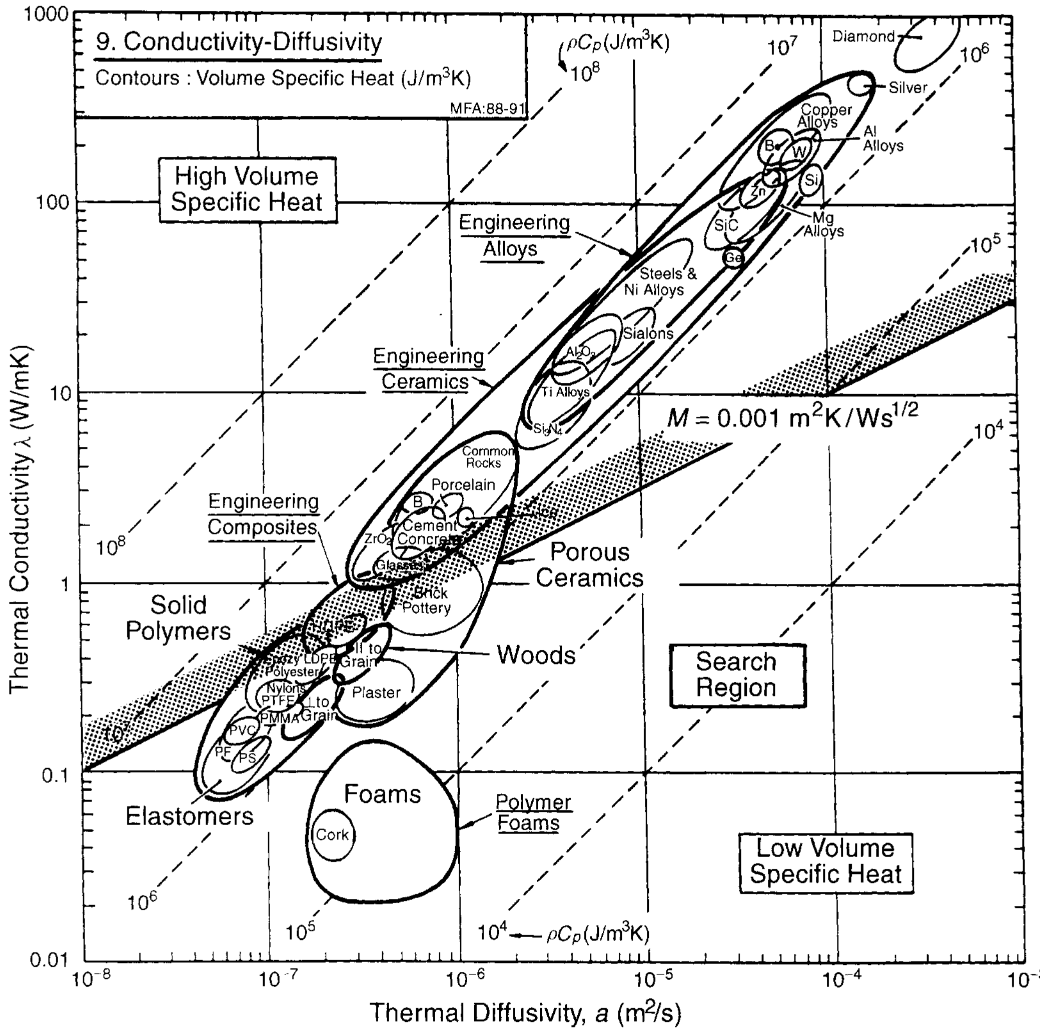

Fig. 6.36 Materials for kiln walls. Low density, porous or foam-like ceramics are the best choice.

Postscript

It is not generally appreciated that, in an efficiently-designed kiln, as much energy goes in heating up the kiln itself as is lost by thermal conduction to the outside environment. It is a mistake to make kiln walls too thick; a little is saved in reduced conduction-loss, but more is lost in the greater heat capacity of the kiln itself.

That, too is the reason that foams are good: they have a low thermal conductivity *and* a low heat capacity. Centrally heated houses in which the heat is turned off at night suffer a cycle like that of the kiln. Here (because T_{max} is lower) the best choice is a polymeric foam, cork or fibreglass (which has thermal properties like those of foams). But as this case study shows — turning the heat off at night doesn't save you as much as you think, because you have to supply the heat capacity of the walls in the morning.

Table 6.36 Materials for energy-efficient kilns

Material	$M = a^{1/2}/\lambda$ ($m^2 K/W s^{1/2}$)	*Thickness, t* (*m*)	*Comment*
Porous ceramics	3×10^{-4}–3×10^{-3}	0.1	The obvious choice: the lower the density, the better the performance.
Solid elastomers	10^{-3}–3×10^{-3}	0.05	Good values of material index. Useful if the wall must be very thin.
Solid polymers	10^{-3}		Limited to temperatures below 150°C.
Polymer foam, Cork	3×10^{-3}–3×10^{-2}	0.09	The highest value of M — hence their use in house insulation. Limited to temperatures below 150°C.
Woods	3×10^{-3}	0.07	The boiler of Stevenson's 'Rocket' was insulated with wood.
Fibreglass	10^{-2}	0.1	Thermal properties comparable with polymer foams; usable to 200°C.

Further reading

Holman, J.P. (1981) *Heat Transfer* 5th edition, McGraw-Hill, New York.

Related case studies

Case Study 6.17: Insulation for short-term isothermal containers
Case Study 6.19: Materials for passive solar heating

6.19 Materials for passive solar heating

There are a number of schemes for capturing solar energy for home heating: solar cells, liquid filled heat exchangers, and solid heat reservoirs. The simplest of these is the heat-storing wall: a thick wall, the outer surface of which is heated by exposure to direct sunshine during the day, and from which heat is extracted at night by blowing air over its inner surface (Figure 6.37). An essential of such a scheme is that the time-constant for heat flow through the wall be about 12 hours; then the wall first warms on the inner surface roughly 12 hours after the sun first warms the outer one, giving out at night what it took in during the day. We will suppose that, for architectural reasons, the wall must not be more than 0.5 m thick. What materials maximize the thermal energy captured by the wall while retaining a heat-diffusion time of up to 12 hours? Table 6.37 summarizes the requirements.

The model

The heat content, Q, per unit area of wall, when heated through a temperature interval ΔT gives the objective function

$$Q = w\rho C_p \Delta T \tag{6.57}$$

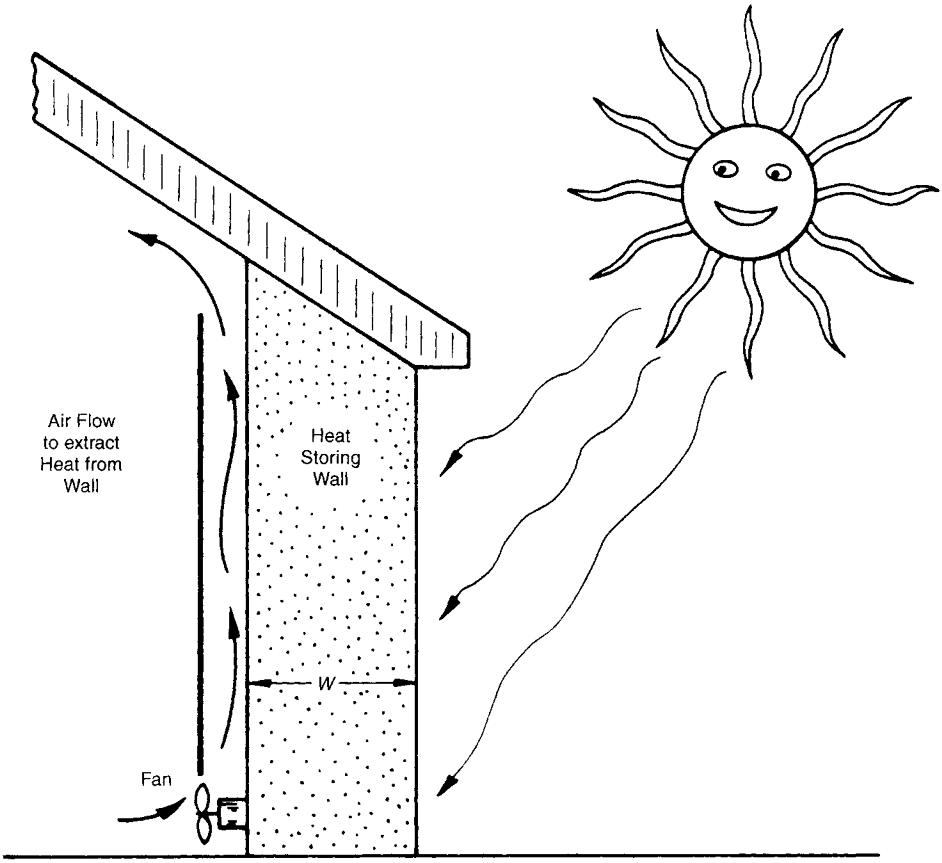

Fig. 6.37 A heat-storing wall. The sun shines on the outside during the day; heat is extracted from the inside at night. The heat diffusion-time through the wall must be about 12 hours.

Table 6.37 Design requirements for passive solar heating

Function	Heat-storing medium
Objective	Maximize thermal energy stored per unit material cost
Constraints	(a) Heat diffusion time through wall $t \approx 12$ hours (b) Wall thickness ≤ 0.5 m (c) Adequate working temperature $T_{max} > 100°C$

where w is the wall thickness, and ρC_p is the volumetric specific heat (the density ρ times the specific heat C_p). The 12-hour time constant is a constraint. It is adequately estimated by the approximation (see Appendix A, 'Useful Solutions')

$$w = \sqrt{2at} \tag{6.58}$$

where a is the thermal diffusivity and t the time. Eliminating the free variable w gives

$$Q = \sqrt{2t}\Delta T a^{1/2} \rho C_p \tag{6.59}$$

or, using the fact that $a = \lambda/\rho C_p$ where λ is the thermal conductivity,

$$Q = \sqrt{2t}\Delta T\lambda/a^{1/2}$$

The heat capacity of the wall is maximized by choosing material with a high value of

$$\boxed{M_1 = \frac{\lambda}{a^{1/2}}} \tag{6.60}$$

— it is the inverse of the index of Case Study 6.17. The restriction on thickness w requires (from equation 6.58) that

$$a \leq \frac{w^2}{2t}$$

with $w \leq 0.5$ m and $t = 12$ hours (4×10^4 s), we obtain a material limit

$$\boxed{M_2 = a \leq 3 \times 10^{-6}\,\text{m}^2/\text{s}}$$

The selection

Figure 6.38 shows Chart 9 (thermal conductivity plotted against thermal diffusivity) with M_1 and M_2 plotted on it. It identifies the group of materials, listed in Table 6.38: they maximize M_1 while meeting the constraint expressed by M_2. Solids are good; porous materials and foams (often used in walls) are not.

Postscript

All this is fine, but what of cost? If this scheme is to be used for housing, cost is an important consideration. The relative costs per unit volume, read from Chart 14 (Figure 4.15), are listed in Table 6.38 — it points to the selection of cement, concrete and brick.

Table 6.38 Materials for passive solar heat storage

Material	$M_1 = \lambda/a^{1/2}$ *(W s$^{1/2}$/m^{2}K)*	*Relative Cost (Mg/m^3)*	*Comment*
Cement		0.5	The right choice
Concrete	3×10^{-3}	0.35	depending on availability
Common rocks		1.0	and cost.
Glass	3×10^3	10	Good M_1; transmits visible radiation.
Brick	10^3	0.8	Less good than concrete.
HDPE	10^3	3	Too expensive.
Ice	3×10^3	0.1	Attractive value of M; pity it melts at 0°C.

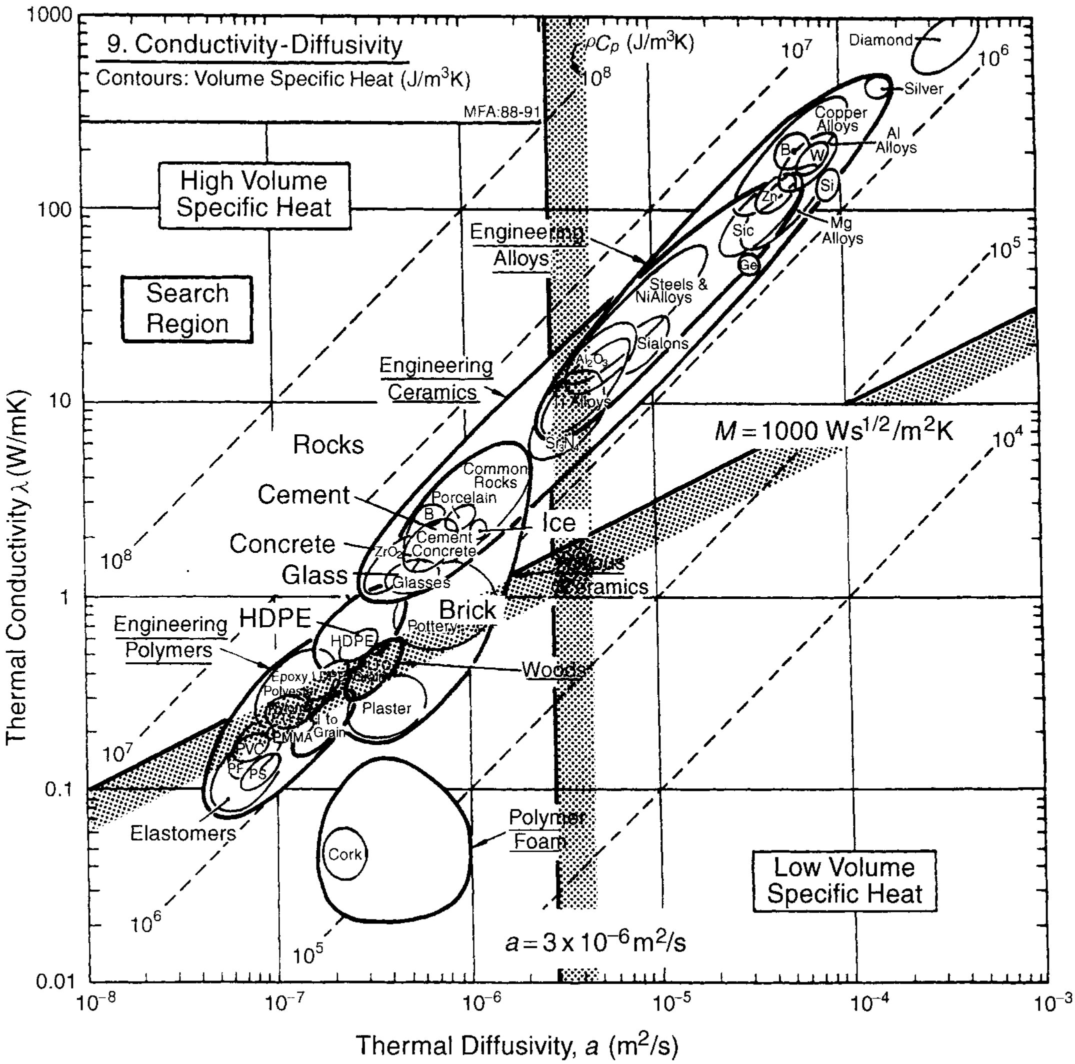

Fig. 6.38 Materials for heat-storing walls. Cement, concrete and stone are practical choices; brick is less good.

If minimizing cost, rather than maximizing Q, were the primary design goal, the model changes. The cost per unit area, C, of the wall is

$$C = w\rho C_m$$

where C_m is the cost per kg of the wall material. The requirement of the 12-hour time-constant remains the same as before (equation (6.58)). Eliminating w gives

$$C = (t)^{1/2}(a^{1/2}\rho C_m)$$

We now wish to maximize

$$M_3 = (a^{1/2}\rho C_m)^{-1} \tag{6.61}$$

This is a new index, one not contained in Figure 6.38, and there is no chart for making the selection. Software, described in Chapter 5, allows a chart to be constructed for use with any material index. Running this software identifies cement, concrete and ice as the cheapest candidates.

Ice appears in both selections. Here is an example of a forgotten constraint. If a material is to be used in a given temperature range, its maximum use temperature, T_{max}, must lie above it. Restricting the selection to materials with $T_{max} > 100°C$ eliminates ice.

Related case studies

Case Study 6.17: Insulation for short-term isothermal containers
Case Study 6.18: Energy-efficient kiln walls

6.20 Materials to minimize thermal distortion in precision devices

The precision of a measuring device, like a sub-micrometer displacement gauge, is limited by its stiffness and by the dimensional change caused by temperature gradients. Compensation for elastic deflection can be arranged; and corrections to cope with thermal expansion are possible too — provided the device is at a uniform temperature. *Thermal gradients* are the real problem: they cause a change of shape — that is, a distortion of the device — for which compensation is not possible. Sensitivity to *vibration* is also a problem: natural excitation introduces noise and thus imprecision into the measurement. So it is permissible to allow expansion in precision instrument design, provided distortion does not occur (Chetwynd, 1987). Elastic deflection is allowed, provided natural vibration frequencies are high.

What, then, are good materials for precision devices? Table 6.39 lists the requirements.

The model

Figure 6.39 shows, schematically, such a device: it consists of a force loop, an actuator and a sensor. We aim to choose a material for the force loop. It will, in general, support heat sources: the fingers of the operator of the device in the figure, or, more usually, electrical components which generate heat. The relevant material index is found by considering the simple case of one-dimensional heat flow through a rod insulated except at its ends, one of which is at ambient and the other connected

Table 6.39 Design requirements for precision devices

Function	Force loop (frame) for precision device
Objective	Maximize positional accuracy (minimize distortion)
Constraints	(a) Must tolerate heat flux (b) Must tolerate vibration

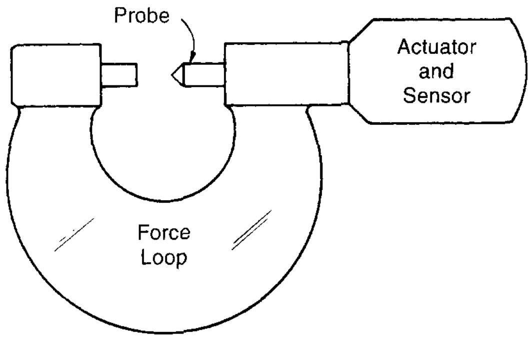

Fig. 6.39 A schematic of a precision measuring device. Super-accurate dimension-sensing devices include the atomic-force microscope and the scanning tunnelling microscope.

to the heat source. In the steady state, Fourier's law is

$$q = -\lambda \frac{dT}{dx} \tag{6.67}$$

where q is heat input per unit area, λ is the thermal conductivity and dT/dx is the resulting temperature gradient. The strain is related to temperature by

$$\varepsilon = \alpha(T_o - T) \tag{6.68}$$

where α is the thermal conductivity and T_o is ambient temperature. The distortion is proportional to the gradient of the strain:

$$\frac{d\varepsilon}{dx} = \frac{\alpha\, dT}{dx} = \left(\frac{\alpha}{\lambda}\right) q \tag{6.69}$$

Thus for a given geometry and heat flow, the distortion $d\varepsilon/dx$ is minimized by selecting materials with large values of the index

$$\boxed{M_1 = \frac{\lambda}{\alpha}}$$

The other problem is vibration. The sensitivity to external excitation is minimized by making the natural frequencies of the device as high as possible. The flexural vibrations have the lowest frequencies; they are proportional to

$$\boxed{M_2 = \frac{E^{1/2}}{\rho}}$$

A high value of this index will minimize the problem. Finally, of course, the device must not cost too much.

The selection

Chart 10 (Figure 6.40) shows the expansion coefficient, α, plotted against the thermal conductivity, λ. Contours show constant values of the quantity λ/α. A search region is isolated by the line $\lambda/\alpha = 10^7$ W/m, giving the shortlist of Table 6.40. Values of $M_2 = E^{1/2}/\rho$ read from Chart 1 (Figure 4.2) are included in the table. Diamond is outstanding, but practical only for very small devices. The metals, except for beryllium, are disadvantaged by having high densities and thus poor values of M_2. The best choice is silicon, available in large sections, with high purity. Silicon carbide is an alternative.

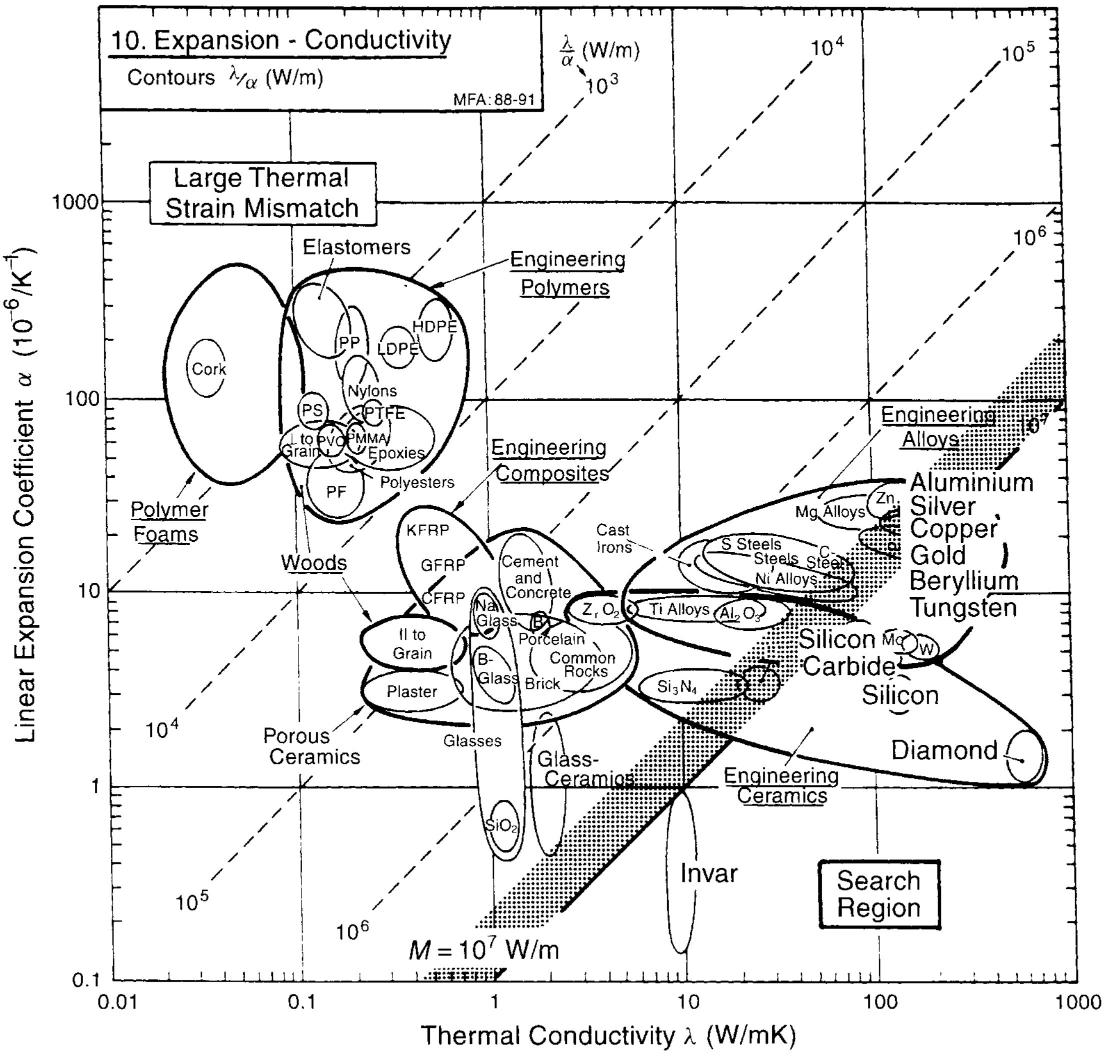

Fig. 6.40 Materials for precision measuring devices. Metals are less good than ceramics because they have lower vibration frequencies. Silicon may be the best choice.

Table 6.40 Materials to minimize thermal distortion

Material	$M_1 = \lambda/\alpha$ *(W/m)*	$M_2 = E^{1/2}/\rho$ *(GPa$^{1/2}$/(Mg/m^3))*	*Comment*
Diamond	5×10^8	8.6	Outstanding M_1 and M_2; expensive.
Silicon	4×10^7	6.0	Excellent M_1 and M_2; cheap.
Silicon carbide	2×10^7	6.2	Excellent M_1 and M_2; potentially cheap.
Beryllium	10^7	9	Less good than silicon or SiC.
Aluminium	10^7	3.1	Poor M_1, but very cheap.
Silver	2×10^7	1.0	High density
Copper	2×10^7	1.3	gives poor
Gold	2×10^7	0.6	value of M_2.
Tungsten	3×10^7	1.1	Better than copper, silver or
Molybdenum	2×10^7	1.3	gold, but less good than
Invar	3×10^7	1.4	silicon, SiC, diamond.

Postscript

Nano-scale measuring and imaging systems present the problem analysed here. The atomic-force microscope and the scanning-tunnelling microscope both support a probe on a force loop, typically with a piezo-electric actuator and electronics to sense the proximity of the probe to the test surface. Closer to home, the mechanism of a video recorder and that of a hard disk drive qualify as precision instruments; both have an actuator moving a sensor (the read head) attached, with associated electronics, to a force loop. The materials identified in this case study are the best choice for force loop.

Further reading

Chetwynd, D.G. (1987) *Precision Engineering*, **9**(1), 3.
Cebon, D. and Ashby, M.F. (1994) *Meas. Sci. and Technol.*, **5**, 296.

Related case studies

Case Study 6.3: Mirrors for large telescopes
Case Study 6.17: Insulation for short-term isothermal containers
Case Study 6.21: Ceramic valves for taps

6.21 Ceramic valves for taps

Few things are more irritating than a dripping tap. Taps drip because the rubber washer is worn, or the brass seat is pitted by corrosion, or both. Could an alternative choice of materials overcome the problem? Ceramics wear well, and they have excellent corrosion resistance in both pure and salt water. How about a tap with a ceramic valve and seat?

Figure 6.41 shows a possible arrangement. Two identical ceramic discs are mounted one above the other, spring-loaded so that their faces, polished to a tolerance of 0.5 μm, are in contact. The

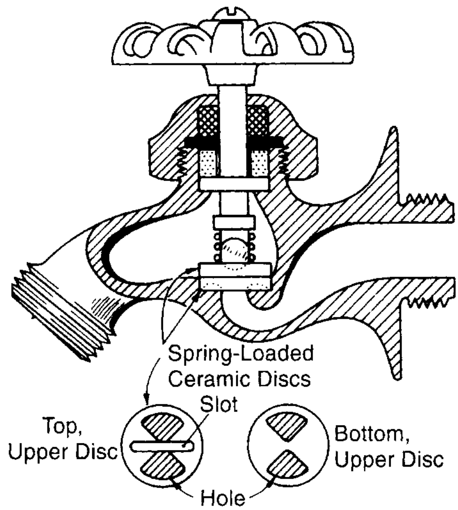

Fig. 6.41 A design for a ceramic valve: two ceramic discs, spring loaded, have holes which align when the tap is turned on.

outer face of each has a slot which registers it, and allows the upper disc to be rotated through 90° (1/4 turn). In the 'off' position the holes in the upper disc are blanked off by the solid part of the lower one; in the 'on' position the holes are aligned. Normal working loads should give negligible wear in the expected lifetime of the tap. Taps with vitreous alumina valves are now available. The manufacturers claim that they do not need any servicing and that neither sediment nor hard water can damage them.

But do they live up to expectation? As cold-water taps they perform well. But as hot-water taps, there is a problem: the discs sometimes crack. The cracking appears to be caused by thermal shock or by thermal mismatch between disc and tap body when the local temperature suddenly changes (as it does when the tap is turned on). Would another ceramic be better? Table 6.41 lists the requirements.

The model

When the water flowing over the ceramic disc suddenly changes in temperature (as it does when you run the tap) the surface temperature of the disc changes suddenly by ΔT. The thermal strain of the surface is proportional to $\alpha\Delta T$ where α is the linear expansion coefficient; the constraint

Table 6.41 Design requirements for ceramic valves for taps

Function	Ceramic valve
Objective	Maximize life
Constraints	(a) Must withstand thermal shock (b) High hardness to resist wear (c) No corrosion in tap water

exerted by the interior of the disc generates a thermal stress

$$\sigma \approx E\alpha\Delta T \tag{6.72}$$

If this exceeds the tensile strength of the ceramic, fracture will result. We require, for damage-free operation, that

$$\sigma \leq \sigma_t$$

The safe temperature interval ΔT is therefore maximized by choosing materials with large values of

$$\boxed{M_1 = \frac{\sigma_t}{E\alpha}}$$

This self-induced stress is one possible origin for valve failures. Another is the expansion mismatch between the valve and the metal components with which it mates. The model for this is almost the same; it is simply necessary to replace the thermal expansion coefficient of the ceramic, α, by the difference, $\Delta\alpha$, between the ceramic and the metal.

The selection

The thermal shock resistance of materials is summarized by Chart 12, reproduced as Figure 6.42. From it we see that alumina ceramics (particularly those containing a high proportion of glassy phases) have poor thermal shock resistance: a sudden temperature change of 80°C can crack them, and mechanical loading makes this worse.

The answer is to select a ceramic with a greater resistance to thermal shock. Almost any engineering ceramic is better — notably zirconia, silicon nitride, silicon carbide or sialon (Table 6.42).

Postscript

So ceramic valves for taps appear to be viable. The gain is in service life: the superior wear and corrosion resistance of the ceramic reduce both to a negligible level. But the use of ceramics and metals together raises problems of matching which require careful redesign, and informed material selection procedures.

Related case studies

Case Study 6.20: Minimizing distortion in precision devices

Table 6.42 Materials for ceramic valves

Material	*Comment*
Aluminas, Al_2O_3 with glass	Cheap, but poor thermal shock resistance.
Zirconia, ZrO_2 Silicon carbides, SiC Silicon nitrides, Si_3N_4 Sialons Mullites	All are hard, corrosion resistant in water and most aqueous solutions, and have better thermal shock resistance than aluminas.

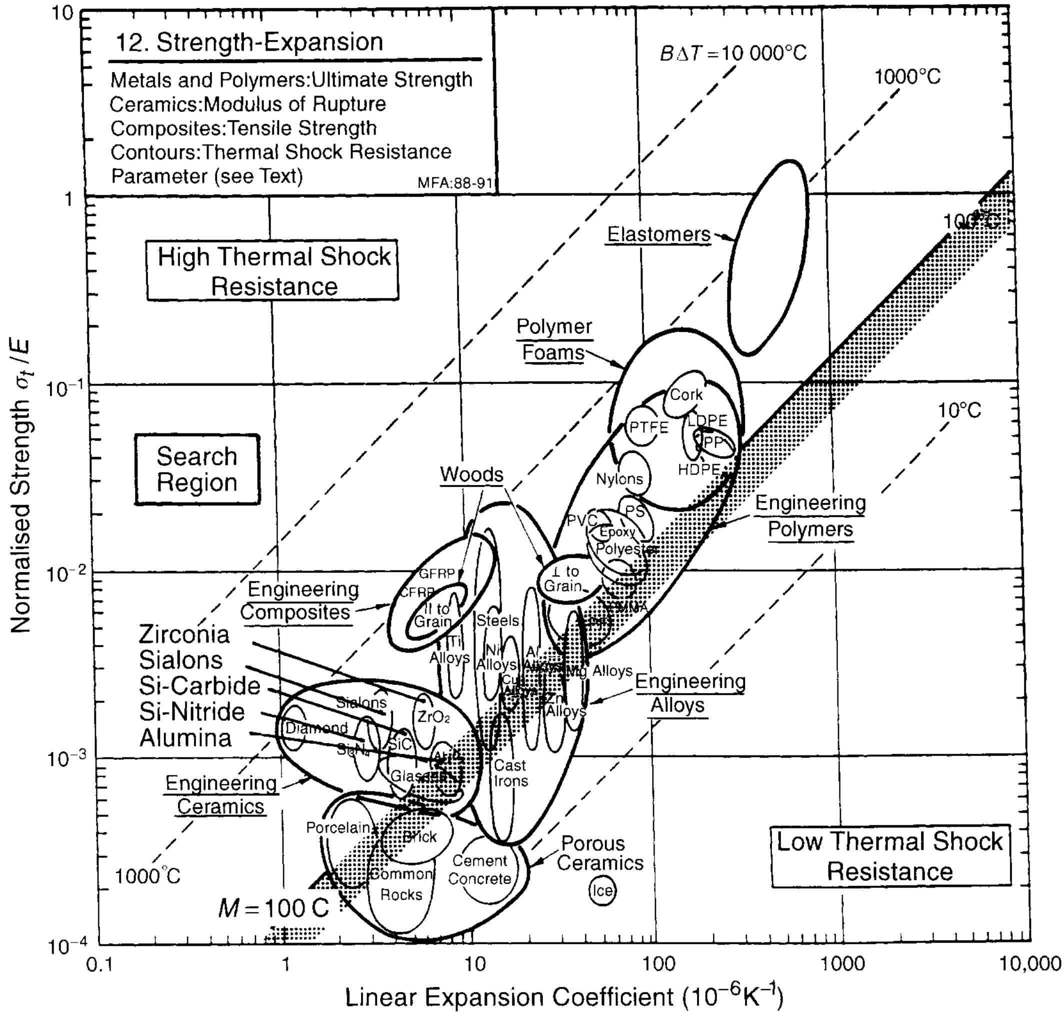

Fig. 6.42 The selection of a material for the ceramic valve of a tap. A ceramic with good thermal shock resistance is desirable.

6.22 Nylon bearings for ships' rudders

Rudder bearings of ships (Figure 6.43) operate under the most unpleasant conditions. The sliding speed is low, but the bearing pressure is high and adequate lubrication is often difficult to maintain. The rudder lies in the wake of the propeller, which generates severe vibration and consequent fretting. Sand and wear debris tend to get trapped between the bearing surfaces. Add to this the environment — aerated salt water — and you can see that bearing design is something of a challenge (Table 6.43).

Ship bearings are traditionally made of bronze. The wear resistance of bronzes is good, and the maximum bearing pressure (important here) is high. But, in sea water, galvanic cells are set up

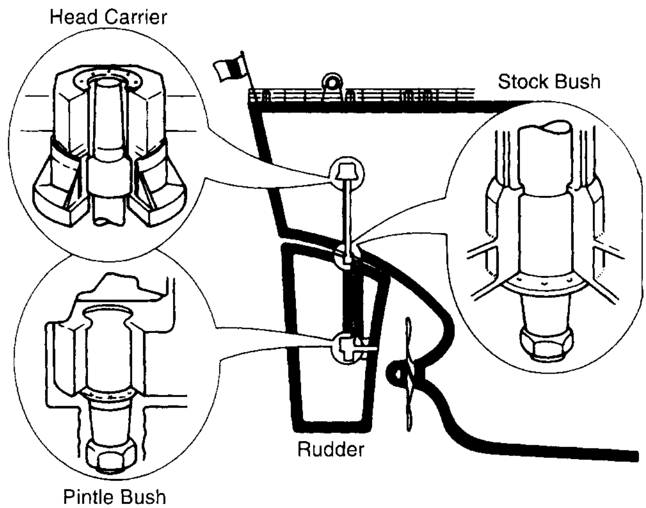

Fig. 6.43 A ship's rudder and its bearings.

Table 6.43 Design requirements for rudder bearings

Function	Sliding bearing
Objective	Maximize life
Constraints	(a) Wear resistant with water lubrication (b) Resist corrosion in sea water (c) High damping desirable

between the bronze and any other metal to which it is attached by a conducting path (no matter how remote), and in a ship such connections are inevitable. So galvanic corrosion, as well as abrasion by sand, is a problem. Is there a better choice than bronze?

The model

We assume (reasonably) that the bearing *force* F is fixed by the design of the ship. The bearing *pressure*, P, can be controlled by changing the area A of the bearing surface:

$$P \propto \frac{F}{A}$$

This means that we are free to choose a material with a lower maximum bearing pressure provided the length of the bearing itself is increased to compensate. With this thought in mind, we seek a bearing material which will not corrode in salt water and can function without full lubrication.

The selection

Figure 6.44 shows Chart 16, the chart of wear-rate constant, k_a, and hardness, H. The wear-rate, W, is given by equation (4.29), which, repeated, is

$$\Omega = k_a P = C\left(\frac{P}{P_{\max}}\right) k_a H$$

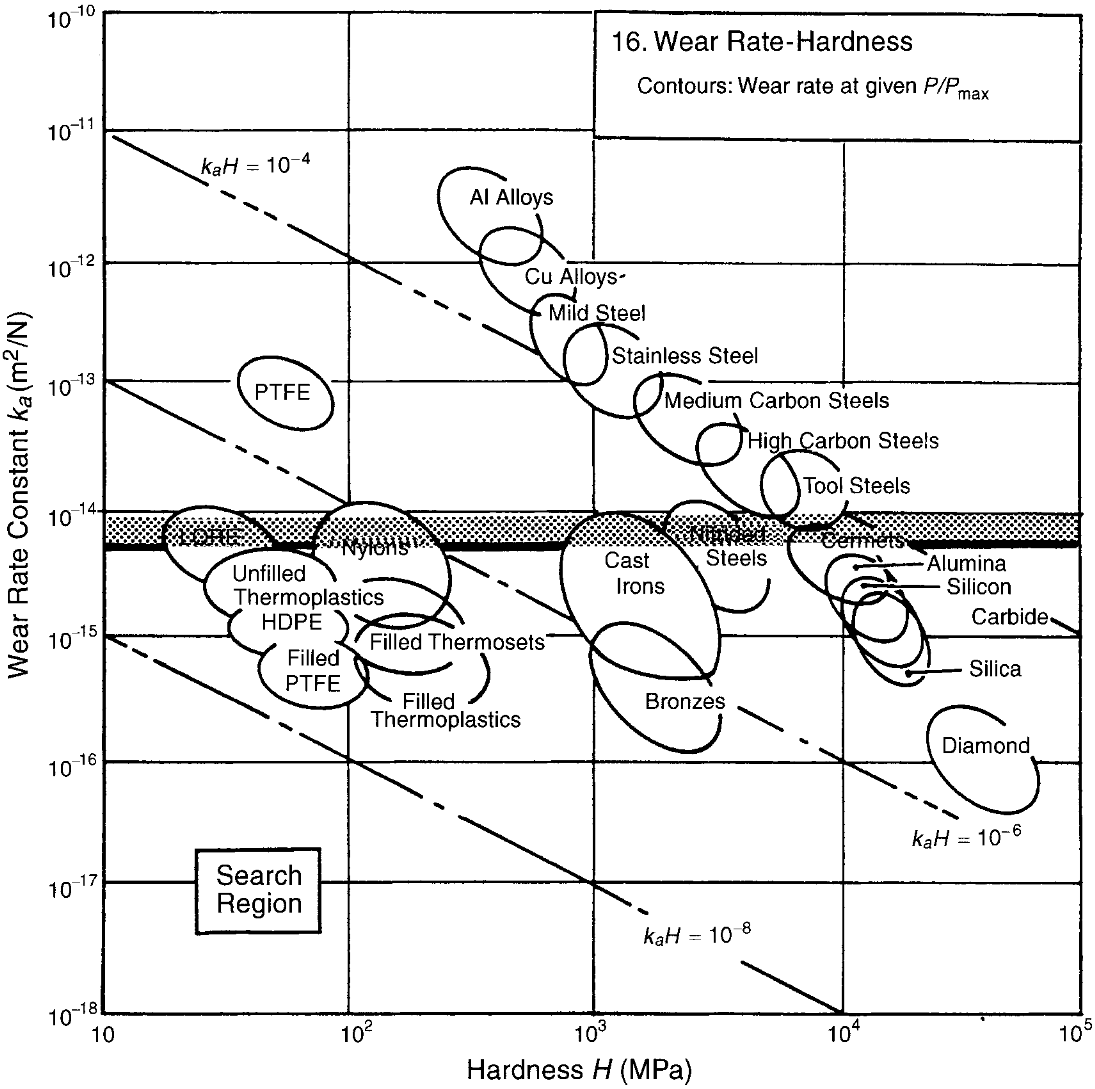

Fig. 6.44 Materials for rudder bearings. Wear is very complex, so the chart gives qualitative guidance only. It suggests that polymers such as nylon or filled or reinforced polymers might be an alternative to bronze provided the bearing area is increased appropriately.

where C is a constant, P is the bearing pressure, P_{max} the maximum allowable bearing pressure for the material, and H is its hardness. If the bearing is not re-sized when a new material is used, the bearing pressure P is unchanged and the material with the lowest wear-rate is simply that with the smallest value of k_a. Bronze performs well, but filled thermoplastics are nearly as good and have superior corrosion resistance in salt water. If, on the other hand, the bearing is re-sized so that it operates at a set fraction of P_{max} (0.5, say), the material with the lowest wear-rate is that with the smallest value of k_aH. Here polymers are clearly superior. Table 6.44 summarizes the conclusions.

Table 6.44 Materials for rudder bearings

Material	*Comment*
PTFE, polyethylenes polypropylenes	Low friction and good wear resistance at low bearing pressures.
Glass-reinforced PTFE, polyethylenes and polypropylenes	Excellent wear and corrosion resistance in sea water. A viable alternative to bronze if bearing pressures are not too large.
Silica, alumina, magnesia	Good wear and corrosion resistance but poor impact properties and very low damping.

Postscript

Recently, at least one manufacturer of marine bearings has started to supply cast nylon 6 bearings for large ship rudders. The makers claim just the advantages we would expect from this case study:

(a) wear and abrasion resistance with water lubrication is improved;
(b) deliberate lubrication is unnecessary;
(c) corrosion resistance is excellent;
(d) the elastic and damping properties of nylon 6 protect the rudder from shocks (see Chart 7: Damping/modulus);
(e) there is no fretting.

Further, the material is easy to handle and install, and is inexpensive to machine.

Figure 6.44 suggests that a filled polymer or composite might be even better. Carbon-fibre filled nylon has better wear resistance than straight nylon, but it is less tough and flexible, and it does not damp vibration as effectively. As in all such problems, the best material is the one which comes closest to meeting *all* the demands made on it, not just the primary design criterion (in this case, wear resistance). The suggestion of the chart is a useful one, worth a try. It would take sea-tests to tell whether it should be adopted.

Related case studies

Case Study 6.21: Ceramic valves for taps

6.23 Summary and conclusions

The case studies of this chapter illustrate how the choice of material is narrowed from the initial, broad, menu to a small subset which can be tried, tested, and examined further. Most designs make certain non-negotiable demands on a material: it must withstand a temperature greater than T, it must resist corrosive fluid F, and so forth. These constraints narrow the choice to a few broad classes of material. The choice is narrowed further by seeking the combination of properties which maximize performance (combinations like $E^{1/2}/\rho$) or maximize safety (combinations like K_{Ic}/σ_f). These, plus economics, isolate a small subset of materials for further consideration.

The final choice between these will depend on more detailed information on their properties, considerations of manufacture, economics and aesthetics. These are discussed in the chapters which follow.

6.24 Further reading

Compilations of case studies starting with the full materials menu

A large compilation of case studies, including many of those given here but with more sophisticated, computer-based selections, is to be found in

Ashby, M.F. and Cebon, D. (1996) *Case Studies in Materials Selection*, published by Granta Design, Trumpington Mews, 40B High Street, Trumpington CB2 2LS, UK.

General texts

The texts listed below give detailed case studies of materials selection. They generally assume that a shortlist of candidates is already known and argue their relative merits, rather than starting with a clean slate, as we do here.

Charles, J.A., Crane, F.A.A. and Furness J.A.G. (1987) *Selection and Use of Engineering Materials*, 3rd edition, Butterworth-Heinemann, Oxford.

Dieter, G.E. (1991) *Engineering Design, A Materials and Processing Approach*, 2nd edition, McGraw-Hill, New York.

Lewis, G. (1990) *Selection of Engineering Materials*, Prentice-Hall, Englewood Cliffs, NJ.